CHAPTER 4
Trigonometry

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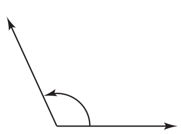
Section 4.1 Radian and Degree Measure

You should know the following basic facts about angles, their measurement, and their applications.

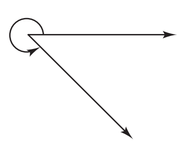
- Types of Angles:
  - (a) Acute: Measure between 0° and 90°.
  - (b) Right: Measure 90°.
  - (c) Obtuse: Measure between 90° and 180°.
  - (d) Straight: Measure 180°.
- α and β are complementary if $\alpha + \beta = 90°$. They are supplementary if $\alpha + \beta = 180°$.
- Two angles in standard position that have the same terminal side are called coterminal angles.
- To convert degrees to radians, use $1° = \pi/180$ radians.
- To convert radians to degrees, use 1 radian $= (180/\pi)°$.
- $1' =$ one minute $= 1/60$ of $1°$.
- $1'' =$ one second $= 1/60$ of $1' = 1/3600$ of $1°$.
- The length of a circular arc is $s = r\theta$ where θ is measured in radians.
- Linear speed $= \dfrac{\text{arc length}}{\text{time}} = \dfrac{s}{t}$
- Angular speed $= \theta/t = s/rt$

Vocabulary Check

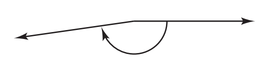
1. Trigonometry
2. angle
3. coterminal
4. radian
5. acute; obtuse
6. complementary; supplementary
7. degree
8. linear
9. angular
10. $A = \frac{1}{2}r^2\theta$

1. 

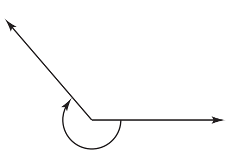
The angle shown is approximately 2 radians.

2. 

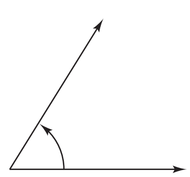
The angle shown is approximately 5.5 radians.

3. 

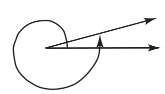
The angle shown is approximately -3 radians.

4. 

The angle shown is approximately -4 radians.

5. 

The angle shown is approximately 1 radian.

6. 

The angle shown is approximately 6.5 radians.

7. (a) Since $0 < \frac{\pi}{5} < \frac{\pi}{2}$; $\frac{\pi}{5}$ lies in Quadrant I.

(b) Since $\pi < \frac{7\pi}{5} < \frac{3\pi}{2}$; $\frac{7\pi}{5}$ lies in Quadrant III.

8. (a) Since $\pi < \frac{11\pi}{8} < \frac{3\pi}{2}$; $\frac{11\pi}{8}$ lies in Quadrant III.

(b) Since $\pi < \frac{9\pi}{8} < \frac{3\pi}{2}$; $\frac{9\pi}{8}$ lies in Quadrant III.

9. (a) Since $-\frac{\pi}{2} < -\frac{\pi}{12} < 0$; $-\frac{\pi}{12}$ lies in Quadrant IV.

(b) Since $-\pi < -2 < -\frac{\pi}{2}$; -2 lies in Quadrant III.

10. (a) Since $-\frac{\pi}{2} < -1 < 0$; -1 lies in Quadrant IV.

(b) Since $-\frac{3\pi}{2} < -\frac{11\pi}{9} < -\pi$, $-\frac{11\pi}{9}$ lies in Quadrant II.

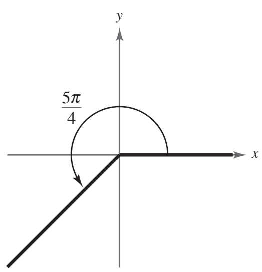
11. (a) Since $\pi < 3.5 < \frac{3\pi}{2}$; 3.5 lies in Quadrant III.

(b) Since $\frac{\pi}{2} < 2.25 < \pi$; 2.25 lies in Quadrant II.

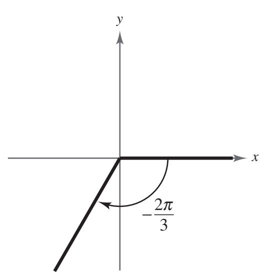
12. (a) Since $\frac{3\pi}{2} < 6.02 < 2\pi$; 6.02 lies in Quadrant IV.

(b) Since $-\frac{3\pi}{2} < -4.25 < -\pi$; 4.25 lies in Quadrant II.

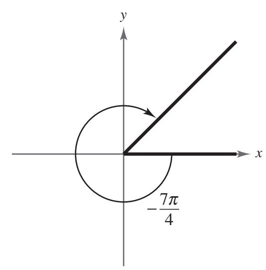
13. (a) $\frac{5\pi}{4}$



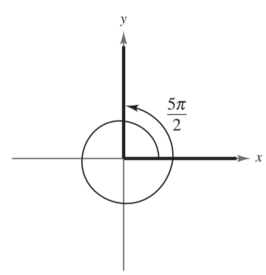
(b) $-\frac{2\pi}{3}$



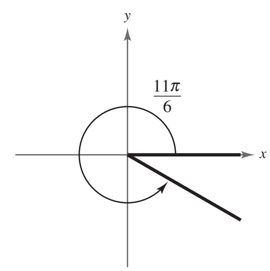
14. (a) $-\frac{7\pi}{4}$



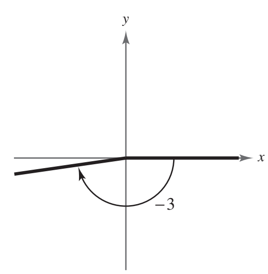
(b) $\frac{5\pi}{2}$



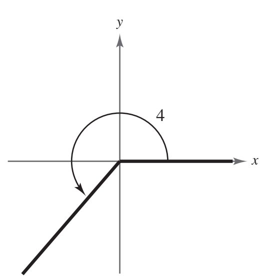
15. (a) $\frac{11\pi}{6}$



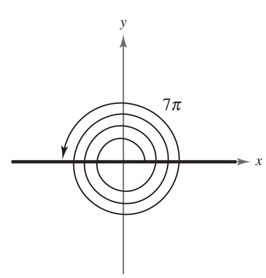
(b) -3



16. (a) 4



(b) 7π



17. (a) Coterminal angles for $\frac{\pi}{6}$

$$\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$$

$$\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$$

(b) Coterminal angles for $\frac{5\pi}{6}$

$$\frac{5\pi}{6} + 2\pi = \frac{17\pi}{6}$$

$$\frac{5\pi}{6} - 2\pi = -\frac{7\pi}{6}$$

18. (a) $\frac{7\pi}{6} + 2\pi = \frac{19\pi}{6}$

$$\frac{7\pi}{6} - 2\pi = -\frac{5\pi}{6}$$

(b) $-\frac{11\pi}{6} + 2\pi = \frac{\pi}{6}$

$$-\frac{11\pi}{6} - 2\pi = -\frac{23\pi}{6}$$

19. (a) Coterminal angles for $\frac{2\pi}{3}$

$$\frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}$$

$$\frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3}$$

(b) Coterminal angles for $\frac{\pi}{12}$

$$\frac{\pi}{12} + 2\pi = \frac{25\pi}{12}$$

$$\frac{\pi}{12} - 2\pi = -\frac{23\pi}{12}$$

20. (a) $-\frac{9\pi}{4} + 2\pi = -\frac{\pi}{4}$

$$-\frac{9\pi}{4} + 4\pi = \frac{7\pi}{4}$$

(b) $-\frac{2\pi}{15} + 2\pi = \frac{28\pi}{15}$

$$-\frac{2\pi}{15} - 2\pi = -\frac{32\pi}{15}$$

21. (a) Complement: $\frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$

Supplement: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

(b) Complement: Not possible, $\frac{3\pi}{4}$ is greater than $\frac{\pi}{2}$.

Supplement: $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$

22. (a) Complement: $\frac{\pi}{2} - \frac{\pi}{12} = \frac{5\pi}{12}$

Supplement: $\pi - \frac{\pi}{12} = \frac{11\pi}{12}$

(b) Complement: Not possible, $\frac{11\pi}{12}$ is greater than $\frac{\pi}{2}$.

Supplement: $\pi - \frac{11\pi}{12} = \frac{\pi}{12}$

23. (a) Complement: $\frac{\pi}{2} - 1 \approx 0.57$

Supplement: $\pi - 1 \approx 2.14$

(b) Complement: Not possible, 2 is greater than $\frac{\pi}{2}$.

Supplement: $\pi - 2 \approx 1.14$

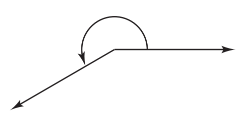
24. (a) Complement: Not possible, 3 is greater than $\frac{\pi}{2}$.

Supplement: $\pi - 3 \approx 0.14$

(b) Complement: $\frac{\pi}{2} - 1.5 \approx 0.07$

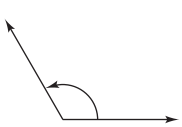
Supplement: $\pi - 1.5 \approx 1.64$

25.



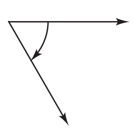
The angle shown is approximately 210°.

26.



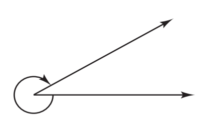
The angle shown is approximately 120°.

27.



The angle shown is approximately −60°.

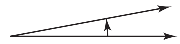
28.



The angle shown is approximately −330°.

29. 

The angle shown is approximately 165°.

30. 

The angle shown is approximately 10°.

31. (a) Since $90° < 130° < 180°$, $130°$ lies in Quadrant II.

(b) Since $270° < 285° < 360°$, $285°$ lies in Quadrant IV.

32. (a) Since $0° < 8.3° < 90°$, $8.3°$ lies in Quadrant I.

(b) Since $180° < 257° 30' < 270°$, $257° 30'$ lies in Quadrant III.

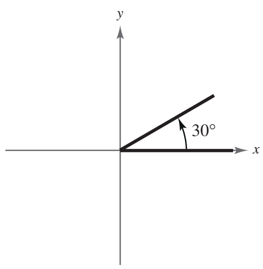
33. (a) Since $-180° < -132° 50' < -90°$, $-132° 50'$ lies in Quadrant III.

(b) Since $-360° < -336° < -270°$, $-336°$ lies in Quadrant I.

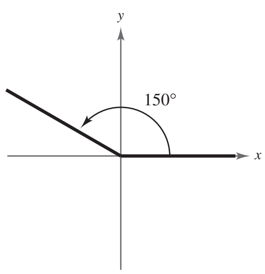
34. (a) Since $-270° < -260° < -180°$, $-260°$ lies in Quadrant II.

(b) Since $-90° < -3.4° < 0°$, $-3.4°$ lies in Quadrant IV.

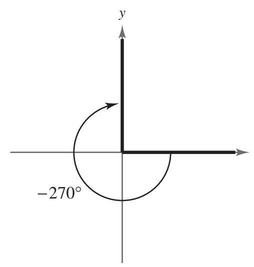
35. (a) $30°$



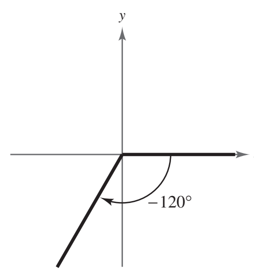
(b) $150°$



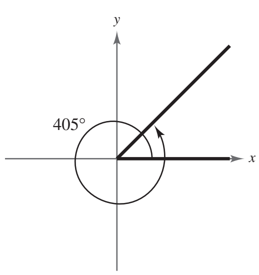
36. (a) $-270°$



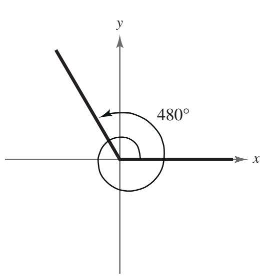
(b) $-120°$



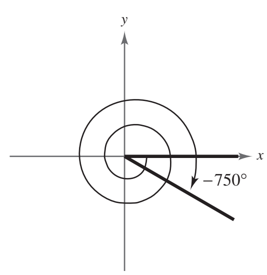
37. (a) $405°$



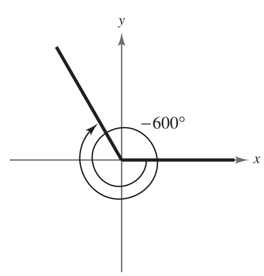
(b) $480°$



38. (a) $-750°$



(b) $-600°$



39. (a) Coterminal angles for $45°$

$45° + 360° = 405°$

$45° - 360° = -315°$

(b) Coterminal angles for $-36°$

$-36° + 360° = 324°$

$-36° - 360° = -396°$

40. (a) $120° + 360° = 480°$

$120° - 360° = -240°$

(b) $-420° + 720° = 300°$

$-420° + 360° = -60°$

41. (a) Coterminal angles for $240°$

$240° + 360° = 600°$

$240° - 360° = -120°$

(b) Coterminal angles for $-180°$

$-180° + 360° = 180°$

$-180° - 360° = -540°$

42. (a) $-420° + 720° = 300°$

$-420° + 360° = -60°$

(b) $230° + 360° = 590°$

$230° - 360° = -130°$

43. (a) Complement: $90° - 18° = 72°$

Supplement: $180° - 18° = 162°$

(b) Complement: Not possible, $115°$ is greater than $90°$.

Supplement: $1180° - 115° = 65°$

44. (a) Complement: $90° - 3° = 87°$

Supplement: $180° - 3° = 177°$

(b) Complement: $90° - 64° = 26°$

Supplement: $180° - 64° = 116°$

45. (a) Complement: $90° - 79° = 11°$

Supplement: $180° - 79° = 101°$

(b) Complement: Not possible, $150°$ is greater than $90°$.

Supplement: $180° - 150° = 30°$

46. (a) Complement: Not possible, $130°$ is greater than $90°$.

Supplement: $180° - 130° = 50°$

(b) Complement: Not possible, $170°$ is greater than $90°$.

Supplement: $180° - 170° = 10°$

47. (a) $30° = 30\left(\frac{\pi}{180}\right) = \frac{\pi}{6}$

(b) $150° = 150\left(\frac{\pi}{180}\right) = \frac{5\pi}{6}$

48. (a) $315° = 315°\left(\frac{\pi}{180°}\right) = \frac{7\pi}{4}$

(b) $120° = 120°\left(\frac{\pi}{180°}\right) = \frac{2\pi}{3}$

49. (a) $-20° = -20\left(\frac{\pi}{180}\right) = -\frac{\pi}{9}$

(b) $-240° = -240\left(\frac{\pi}{180}\right) = -\frac{4\pi}{3}$

50. (a) $-270° = -270°\left(\frac{\pi}{180°}\right) = -\frac{3\pi}{2}$

(b) $144° = 144°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{5}$

51. (a) $\frac{3\pi}{2} = \frac{3\pi}{2}\left(\frac{180}{\pi}\right)^{\circ} = 270°$

(b) $\frac{7\pi}{6} = \frac{7\pi}{6}\left(\frac{180}{\pi}\right)^{\circ} = 210°$

52. (a) $-\frac{7\pi}{12} = -\frac{7\pi}{12}\left(\frac{180°}{\pi}\right) = -105°$

(b) $\frac{\pi}{9} = \frac{\pi}{9}\left(\frac{180°}{\pi}\right) = 20°$

53. (a) $\frac{7\pi}{3} = \frac{7\pi}{3}\left(\frac{180}{\pi}\right)^{\circ} = 420°$

(b) $-\frac{11\pi}{30} = -\frac{11\pi}{30}\left(\frac{180}{\pi}\right)^{\circ} = -66°$

54. (a) $\frac{11\pi}{6} = \frac{11\pi}{6}\left(\frac{180°}{\pi}\right) = 330°$

(b) $\frac{34\pi}{15} = \frac{34\pi}{15}\left(\frac{180°}{\pi}\right) = 408°$

55. $115° = 115\left(\frac{\pi}{180}\right)$

≈ 2.007 radians

56. $87.4° = 87.4°\left(\frac{\pi}{180°}\right)$

≈ 1.525 radians

57. $-216.35° = -216.35\left(\frac{\pi}{180}\right) \approx -3.776$ radians

58. $-48.27° = -48.27°\left(\frac{\pi}{180°}\right) \approx -0.842$ radians

59. $532° = 532\left(\frac{\pi}{180}\right) \approx 9.285$ radians

60. $345° = 345°\left(\frac{\pi}{180°}\right) \approx 6.021$ radians

61. $-0.83° = -0.83\left(\frac{\pi}{180}\right) \approx -0.014$ radian

62. $0.54° = 0.54°\left(\frac{\pi}{180°}\right) \approx 0.009$ radians

63. $\frac{\pi}{7} = \frac{\pi}{7}\left(\frac{180}{\pi}\right)^{\circ} \approx 25.714°$

64. $\frac{5\pi}{11} = \frac{5\pi}{11}\left(\frac{180°}{\pi}\right) \approx 81.818°$

65. $\frac{15\pi}{8} = \frac{15\pi}{8}\left(\frac{180}{\pi}\right)^{\circ} = 337.500°$

66. $\frac{13\pi}{2} = \frac{13\pi}{2}\left(\frac{180°}{\pi}\right) = 1170.000°$

67. $-4.2\pi = -4.2\pi\left(\frac{180}{\pi}\right)^{\circ}$

$= -756.000°$

68. $4.8\pi = 4.8\pi\left(\frac{180°}{\pi}\right) = 864.000°$

69. $-2 = -2\left(\frac{180}{\pi}\right)^{\circ} \approx -114.592°$

70. $-0.57 = -0.57\left(\frac{180°}{\pi}\right) \approx -32.659°$

71. (a) $54°\,45' = 54° + \left(\frac{45}{60}\right)^{\circ} = 54.75°$

(b) $-128°\,30' = -128° - \left(\frac{30}{60}\right)^{\circ} = -128.5°$

72. (a) $245°10' = 245° + \left(\frac{10}{60}\right)^{\circ} \approx 245° + 0.167° = 245.167°$

(b) $2°12' = 2° + \left(\frac{12}{60}\right)^{\circ} = 2° + 0.2° = 2.2°$

73. (a) $85°\,18'\,30'' = \left(85 + \frac{18}{60} + \frac{30}{3600}\right)^{\circ} \approx 85.308°$

(b) $330°\,25'' = \left(330 + \frac{25}{3600}\right)^{\circ} \approx 330.007°$

74. (a) $-135°\,36'' = -135° - \left(\frac{36}{3600}\right)^{\circ}$

$= -135° - 0.01° = -135.01°$

(b) $-408°\,16'\,20'' = -\left(408° + \left(\frac{16}{60}\right)^{\circ} + \left(\frac{20}{3600}\right)^{\circ}\right)$

$\approx -(408° + 0.2667° + 0.0056°)$

$\approx -408.272°$

75. (a) $240.6° = 240° + 0.6(60)' = 240°\,36'$

(b) $-145.8° = -[145° + 0.8(60')] = -145°\,48'$

76. (a) $-345.12° = -(345° + (0.12)(60'))$

$= -(345° + 7' + 0.2(60''))$

$= -345°\,7'\,12''$

(b) $0.45° = 0° + (0.45)(60') = 0° + 27' = 0°\,27'$

77. (a) $2.5° = 2°\,30'$

(b) $-3.58° = -(3° + (0.58)(60'))$

$= -(3° + 34' + 0.8(60''))$

$= -3°\,34'\,48''$

78. (a) $-0.355° = -(0° + (0.355)(60'))$

$= -(0° + 21' + (0.3)(60''))$

$= -(0° + 21' + 18'') = -0°\,21'\,18''$

(b) $0.7865 = 0° + (0.7865)(60')$

$= 0° + 47' + (0.19)(60'')$

$= 0° + 47' + 11.4'' = 0°\,47'\,11.4''$

79. $s = r\theta$

$6 = 5\theta$

$\theta = \frac{6}{5}$ radians

80. $s = r\theta$

$29 = 10\,\theta$

$\theta = \frac{29}{10}$ radians

81. $s = r\theta$

$32 = 7\theta$

$\theta = \frac{32}{7} = 4\frac{4}{7}$ radians

82. $s = r\theta$

$60 = 75\theta$

$\theta = \frac{60}{75} = \frac{4}{5}$ radians

Because the angle represented is clockwise, this angle is $-\frac{4}{5}$ radians.

83. $s = r\theta$

$6 = 27\theta$

$\theta = \frac{6}{27} = \frac{2}{9}$ radians

84. $r = 14$ feet, $s = 8$ feet

$\theta = \frac{s}{r} = \frac{8}{14} = \frac{4}{7}$ radians

85. $s = r\theta$

$25 = 14.5\theta$

$\theta = \dfrac{25}{14.5} = \dfrac{50}{29}$ radians

86. $r = 80$ kilometers,
$s = 160$ kilometers

$\theta = \dfrac{s}{r} = \dfrac{160}{80} = 2$ radians

87. $s = r\theta$, θ in radians

$s = 15(180)\left(\dfrac{\pi}{180}\right) = 15\pi$ inches

≈ 47.12 inches

88. $r = 9$ feet, $\theta = 60° = \dfrac{\pi}{3}$

$s = r\theta = 9\left(\dfrac{\pi}{3}\right) = 3\pi$ feet

≈ 9.42 feet

89. $s = r\theta$, θ in radians

$s = 3(1) = 3$ meters

90. $r = 20$ centimeters, $\theta = \dfrac{\pi}{4}$

$s = r\theta = 20\left(\dfrac{\pi}{4}\right) = 5\pi$ centimeters

≈ 15.71 centimeters

91. $A = \dfrac{1}{2}r^2\theta$

$A = \dfrac{1}{2}(4)^2\left(\dfrac{\pi}{3}\right) = \dfrac{8\pi}{3}$ square inches

≈ 8.38 square inches

92. $r = 12$ mm, $\theta = \dfrac{\pi}{4}$

$A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(12)^2\left(\dfrac{\pi}{4}\right)$

$= 18\pi\ \text{mm}^2$

$\approx 56.55\ \text{mm}^2$

93. $A = \dfrac{1}{2}r^2\theta$

$A = \dfrac{1}{2}(2.5)^2(225)\left(\dfrac{\pi}{180}\right)$

≈ 12.27 square feet

94. $r = 1.4$ miles, $\theta = 330°$

$A = \dfrac{1}{2}(1.4)^2\left(\dfrac{330°}{180°}\right)\pi = \dfrac{21.56}{12}\pi \approx 5.64$ square miles

95. $\theta = 41°\,15'\,50'' - 32°\,47'\,39''$

$\approx 8.46972° \approx 0.14782$ radian

$s = r\theta \approx 4000(0.14782) \approx 591.3$ miles

96. $r = 4000$ miles

$\theta = 47°\,37'\,18'' - 37°\,47'\,36'' = 9°\,49'\,42''$

≈ 0.1715 radian

$s = r\theta \approx (4000)(0.1715) = 686.2$ miles

97. $\theta = \dfrac{s}{r} = \dfrac{450}{6378} \approx 0.071$ radian $\approx 4.04°$

98. $r = 3189$ kilometers

$s = r\theta$

$400 = 6378\theta$

$\dfrac{400}{6378} = \theta$

$0.062716 \approx \theta$

The difference in latitude is about 0.062716 radians $\approx 3.59°$.

99. $\theta = \dfrac{s}{r} = \dfrac{2.5}{6} = \dfrac{25}{60} = \dfrac{5}{12}$ radian

100. $\theta = \dfrac{s}{r} = \dfrac{24}{5} = 4.8$ radians $= 4.8\left(\dfrac{180°}{\pi}\right) \approx 275°$

101. (a) 65 miles per hour $= \dfrac{65(5280)}{60} = 5720$ feet per minute

The circumference of the tire is $C = 2.5\pi$ feet.

The number of revolutions per minute is
$r = \dfrac{5720}{2.5\pi} \approx 728.3$ revolutions per minute

(b) The angular speed is $\dfrac{\theta}{t}$.

$\theta = \dfrac{5720}{2.5\pi}(2\pi) = 4576$ radians

Angular speed $= \dfrac{4576 \text{ radians}}{1 \text{ minute}} = 4576$ radians per minute

102. Linear velocity for either pulley: $1700(2\pi) = 3400\pi$ inches per minute

(a) Angular speed of motor pulley: $\omega = \dfrac{v}{r} = \dfrac{3400\pi}{1} = 3400\pi$ radians per minute

Angular speed of the saw arbor: $\omega = \dfrac{v}{r} = \dfrac{3400\pi}{2} = 1700\pi$ radians per minute

(b) Revolutions per minute of the saw arbor: $\dfrac{1700\pi}{2\pi} = 850$ revolutions per minute

103. (a) Angular speed $= \dfrac{(5200)(2\pi) \text{ radians}}{1 \text{ minute}}$

$= 10{,}400\pi$ radians per minute

$\approx 32{,}672.56$ radians per minute

(b) Linear speed $= \dfrac{\left(\dfrac{7.25}{2} \text{ in.}\right)\left(\dfrac{1 \text{ ft}}{12 \text{ in.}}\right)(5200)(2\pi)}{1 \text{ minute}}$

$= 3141\dfrac{2}{3}\pi$ feet per minute

≈ 9869.84 feet per minute

104. (a) 4 rpm $= 4(2\pi)$ radians per minute

$= 8\pi$ radians per minute

≈ 25.13 radians per minute

(b) $r = 25$ ft

$\dfrac{r\theta}{t} = 200\pi$ feet per minute

Linear speed $\approx 25(25.13274)$ feet per minute

≈ 628.32 feet per minute

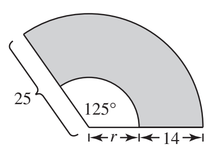
105. (a) $(200)(2\pi) \le$ Angular speed $\le (500)(2\pi)$ radians per minute

Interval: $[400\pi, 1000\pi]$ radians per minute

(b) $(6)(200)(2\pi) \le$ Linear speed $\le (6)(500)(2\pi)$ centimeters per minute

Interval: $[2400\pi, 6000\pi]$ centimeters per minute

106. $A = \dfrac{1}{2}\theta(R^2 - r^2)$

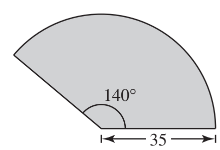


$R = 25$

$r = 25 - 14 = 11$

$A = \dfrac{1}{2}\left(\dfrac{125}{180}\right)\pi \cdot (25^2 - 11^2) = 175\pi \approx 549.8$ square inches

107. $A = \dfrac{1}{2}r^2\theta$



$= \dfrac{1}{2}(35)^2(140°)\left(\dfrac{\pi}{180°}\right)$

$\approx 476.39\pi$ square meters

≈ 1496.62 square meters

108. (a) Arc length of larger sprocket in feet: $s = r\theta$

$$s = \frac{1}{3}(2\pi) = \frac{2\pi}{3} \text{ feet}$$

Therefore, the chain moves $2\pi/3$ feet, as does the smaller rear sprocket. Thus, the angle θ of the smaller sprocket is ($r = 2$ inches $= 2/12$ feet).

$\theta = \dfrac{s}{r} = \dfrac{(2\pi)/3 \text{ feet}}{2/12 \text{ feet}} = 4\pi$ and the arc length of the tire in feet is:

$$s = \theta r$$

$$s = (4\pi)\left(\frac{14}{12}\right) = \frac{14\pi}{3} \text{ feet}$$

$$\text{Speed} = \frac{s}{t} = \frac{(14\pi)/3}{1 \text{ second}} = \frac{14\pi}{3} \text{ feet per second}$$

$\dfrac{14\pi \text{ feet}}{3 \text{ seconds}} \times \dfrac{3600 \text{ seconds}}{1 \text{ hour}} \times \dfrac{1 \text{ mile}}{5280 \text{ feet}} \approx 10$ miles per hour

—CONTINUED—

108. —CONTINUED—

(b) Since the arc length of the tire is $(14\pi)/3$ feet and the cyclist is pedaling at a rate of one revolution per second, we have:

$$\text{Distance} = \left(\frac{14\pi}{3}\,\frac{\text{feet}}{\text{revolutions}}\right)\left(\frac{1\text{ mile}}{5280\text{ feet}}\right)(n\text{ revolutions}) = \frac{7\pi}{7920}n\text{ miles}$$

(c) Distance = Rate · Time

$$= \left(\frac{14\pi}{3}\text{ feet per second}\right)\left(\frac{1\text{ mile}}{5280\text{ feet}}\right)(t\text{ seconds}) = \frac{7\pi}{7920}t\text{ miles}$$

(d) The functions are both linear.

109. False. An angle measure of 4π radians corresponds to two complete revolutions from the initial to the terminal side of an angle.

110. True. If α and β are coterminal angles, then $\alpha = \beta + n(360°)$ where n is an integer. The difference between α and β is $\alpha - \beta = n(360°) = 2\pi n$.

111. False. The terminal side of $-1260°$ lies on the negative x-axis.

112. (a) An angle is in standard position if its vertex is at the origin and its initial side is on the positive x-axis.

(b) A negative angle is generated by a clockwise rotation of the terminal side.

(c) Two angles in standard position with the same terminal sides are coterminal.

(d) An obtuse angle measures between 90° and 180°.

113. Increases, since the linear speed is proportional to the radius.

114. $1\text{ radian} = \left(\dfrac{180}{\pi}\right)^{\circ} \approx 57.3°,$

so one radian is much larger than one degree.

115. The arc length is increasing. In order for the angle θ to remain constant as the radius r increases, the arc length s must increase in proportion to r, as can be seen from the formula $s = r\theta$.

116. The area of a circle is $A = \pi r^2 \implies \pi = \dfrac{A}{r^2}$. The circumference of a circle is $C = 2\pi r$.

$$C = 2\left(\frac{A}{r^2}\right)r$$

$$C = \frac{2A}{r}$$

$$\frac{Cr}{2} = A$$

For a sector, $C = s = r\theta$. Thus, $A = \dfrac{(r\theta)r}{2} = \dfrac{1}{2}\theta r^2$ for a sector.

117. $\dfrac{4}{4\sqrt{2}} = \dfrac{4}{4\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{4\sqrt{2}}{8} = \dfrac{\sqrt{2}}{2}$

118. $\dfrac{5\sqrt{5}}{2\sqrt{10}} = \dfrac{5}{2}\sqrt{\dfrac{5}{10}} = \dfrac{5}{2}\sqrt{\dfrac{1}{2}} = \dfrac{5}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{5\sqrt{2}}{4}$

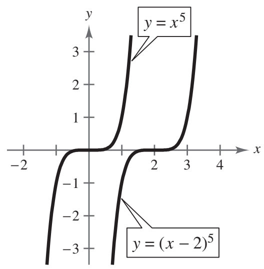
119. $\sqrt{2^2 + 6^2} = \sqrt{4 + 36} = \sqrt{40} = \sqrt{4 \cdot 10} = 2\sqrt{10}$

120. $\sqrt{17^2 - 9^2} = \sqrt{289 - 81}$

$= \sqrt{208} = \sqrt{16 \cdot 13} = 4\sqrt{13}$

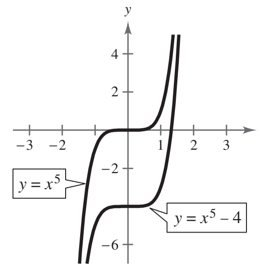
121. $f(x) = (x - 2)^5$

Graph of $y = x^5$ shifted to the right by two units



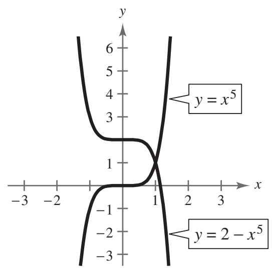
122. $f(x) = x^5 - 4$

Vertical shift four units downward



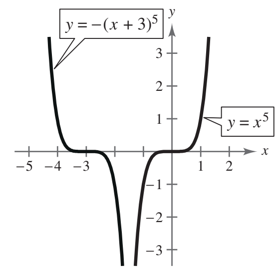
123. $f(x) = 2 - x^5$

Graph of $y = x^5$ reflected in x-axis and shifted upward by two units



124. $f(x) = -(x + 3)^5$

Reflection in the x-axis and a horizontal shift three units to the left



Section 4.2 Trigonometric Functions: The Unit Circle

- You should know the definition of the trigonometric functions in terms of the unit circle. Let t be a real number and (x, y) the point on the unit circle corresponding to t.

$$\sin t = y \qquad \csc t = \frac{1}{y}, \quad y \neq 0$$

$$\cos t = x \qquad \sec t = \frac{1}{x}, \quad x \neq 0$$

$$\tan t = \frac{y}{x}, \quad x \neq 0 \qquad \cot t = \frac{x}{y}, \quad y \neq 0$$

- The cosine and secant functions are even.

$$\cos(-t) = \cos t \qquad \sec(-t) = \sec t$$

- The other four trigonometric functions are odd.

$$\sin(-t) = -\sin t \qquad \csc(-t) = -\csc t$$

$$\tan(-t) = -\tan t \qquad \cot(-t) = -\cot t$$

- Be able to evaluate the trigonometric functions with a calculator.

Vocabulary Check

1. unit circle

2. periodic

3. period

4. odd; even

1. $x = -\frac{8}{17}, y = \frac{15}{17}$

$\sin\theta = y = \frac{15}{17}$ $\quad\csc\theta = \frac{1}{y} = \frac{17}{15}$

$\cos\theta = x = -\frac{8}{17}$ $\quad\sec\theta = \frac{1}{x} = -\frac{17}{8}$

$\tan\theta = \frac{y}{x} = -\frac{15}{8}$ $\quad\cot\theta = \frac{x}{y} = -\frac{8}{15}$

2. $x = \frac{12}{13}, y = \frac{5}{13}$

$\sin\theta = y = \frac{5}{13}$ $\quad\csc\theta = \frac{1}{y} = \frac{13}{5}$

$\cos\theta = x = \frac{12}{13}$ $\quad\sec\theta = \frac{1}{x} = \frac{13}{12}$

$\tan\theta = \frac{y}{x} = \frac{5}{12}$ $\quad\cot\theta = \frac{x}{y} = \frac{12}{5}$

3. $x = \frac{12}{13}, y = -\frac{5}{13}$

$\sin\theta = y = -\frac{5}{13}$ $\quad\csc\theta = \frac{1}{y} = -\frac{13}{5}$

$\cos\theta = x = \frac{12}{13}$ $\quad\sec\theta = \frac{1}{x} = \frac{13}{12}$

$\tan\theta = \frac{y}{x} = -\frac{5}{12}$ $\quad\cot\theta = \frac{x}{y} = -\frac{12}{5}$

4. $x = -\frac{4}{5}, y = -\frac{3}{5}$

$\sin\theta = y = -\frac{3}{5}$ $\quad\csc\theta = \frac{1}{y} = -\frac{5}{3}$

$\cos\theta = x = -\frac{4}{5}$ $\quad\sec\theta = \frac{1}{x} = -\frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{3}{4}$ $\quad\cot\theta = \frac{x}{y} = \frac{4}{3}$

5. $t = \frac{\pi}{4}$ corresponds to $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

6. $t = \frac{\pi}{3}, (x\ y) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

7. $t = \frac{7\pi}{6}$ corresponds to $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

8. $t = \frac{5\pi}{4}, (x, y) = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

9. $t = \frac{4\pi}{3}$ corresponds to $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

10. $t = \frac{5\pi}{3}, (x, y) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

11. $t = \frac{3\pi}{2}$ corresponds to $(0, -1)$.

12. $t = \pi, (x, y) = (-1, 0)$

13. $t = \frac{\pi}{4}$ corresponds to $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$\sin t = y = \frac{\sqrt{2}}{2}$

$\cos t = x = \frac{\sqrt{2}}{2}$

$\tan t = \frac{y}{x} = 1$

14. $t = \frac{\pi}{3}, (x, y) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\tan\frac{\pi}{3} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

15. $t = -\frac{\pi}{6}$ corresponds to $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$\sin t = y = -\frac{1}{2}$

$\cos t = x = \frac{\sqrt{3}}{2}$

$\tan t = \frac{y}{x} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

16. $t = -\frac{\pi}{4}, (x, y) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\tan\left(-\frac{\pi}{4}\right) = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$

17. $t = -\frac{7\pi}{4}$ corresponds to $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$\sin t = y = \frac{\sqrt{2}}{2}$

$\cos t = x = \frac{\sqrt{2}}{2}$

$\tan t = \frac{y}{x} = 1$

18. $t = -\frac{4\pi}{3}, (x, y) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sin\left(-\frac{4\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos\left(-\frac{4\pi}{3}\right) = -\frac{1}{2}$

$\tan\left(-\frac{4\pi}{3}\right) = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$

19. $t = \frac{11\pi}{6}$ corresponds to $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$\sin t = y = -\frac{1}{2}$

$\cos t = x = \frac{\sqrt{3}}{2}$

$\tan t = \frac{y}{x} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

20. $t = \frac{5\pi}{3}, (x, y) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

$\cos \frac{5\pi}{3} = \frac{1}{2}$

$\tan \frac{5\pi}{3} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$

21. $t = -\frac{3\pi}{2}$ corresponds to $(0, 1)$.

$\sin t = y = 1$

$\cos t = x = 0$

$\tan t = \frac{y}{x}$ is undefined.

22. $t = -2\pi, (x, y) = (1, 0)$

$\sin(-2\pi) = 0$

$\cos(-2\pi) = 1$

$\tan(-2\pi) = \frac{0}{1} = 0$

23. $t = \frac{3\pi}{4}$ corresponds to $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$\sin t = y = \frac{\sqrt{2}}{2}$ $\quad$ $\csc t = \frac{1}{y} = \sqrt{2}$

$\cos t = x = -\frac{\sqrt{2}}{2}$ $\quad$ $\sec t = \frac{1}{x} = -\sqrt{2}$

$\tan t = \frac{y}{x} = -1$ $\quad$ $\cot t = \frac{x}{y} = -1$

24. $t = \frac{5\pi}{6}, (x, y) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\sin \frac{5\pi}{6} = \frac{1}{2}$ $\quad$ $\csc \frac{5\pi}{6} = \frac{1}{\sin t} = 2$

$\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$ $\quad$ $\sec \frac{5\pi}{6} = \frac{1}{\cos t} = -\frac{2\sqrt{3}}{3}$

$\tan \frac{5\pi}{6} = \frac{1/2}{-\sqrt{3}/2} = -\frac{\sqrt{3}}{3}$ $\quad$ $\cot \frac{5\pi}{6} = \frac{1}{\tan t} = -\sqrt{3}$

25. $t = -\frac{\pi}{2}$ corresponds to $(0, -1)$.

$\sin t = y = -1$ $\quad$ $\csc t = \frac{1}{y} = -1$

$\cos t = x = 0$ $\quad$ $\sec t = \frac{1}{x}$ is undefined.

$\tan t = \frac{y}{x}$ is undefined. $\quad$ $\cot t = \frac{x}{y} = 0$

26. $t = \frac{3\pi}{2}, (x, y) = (0, -1)$

$\sin \frac{3\pi}{2} = -1$ $\quad$ $\csc \frac{3\pi}{2} = \frac{1}{\sin t} = -1$

$\cos \frac{3\pi}{2} = 0$ $\quad$ $\sec \frac{3\pi}{2}$ is undefined.

$\tan \frac{3\pi}{2}$ is undefined. $\quad$ $\cot \frac{3\pi}{2} = \frac{0}{-1} = 0$

27. $t = \frac{4\pi}{3}$ corresponds to $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$\sin t = y = -\frac{\sqrt{3}}{2}$ $\csc t = \frac{1}{y} = -\frac{2\sqrt{3}}{3}$

$\cos t = x = -\frac{1}{2}$ $\sec t = \frac{1}{x} = -2$

$\tan t = \frac{y}{x} = \sqrt{3}$ $\cot t = \frac{x}{y} = \frac{\sqrt{3}}{3}$

28. $t = \frac{7\pi}{4}, (x, y) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\sin \frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$ $\csc \frac{7\pi}{4} = \frac{1}{\sin t} = -\sqrt{2}$

$\cos \frac{7\pi}{4} = \frac{\sqrt{2}}{2}$ $\sec \frac{7\pi}{4} = \frac{1}{\cos t} = \sqrt{2}$

$\tan \frac{7\pi}{4} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$ $\cot \frac{7\pi}{4} = \frac{1}{\tan t} = -1$

29. $\sin 5\pi = \sin \pi = 0$

30. $\cos 5\pi = \cos \pi = -1$

31. $\cos \frac{8\pi}{3} = \cos \frac{2\pi}{3} = -\frac{1}{2}$

32. $\sin \frac{9\pi}{4} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

33. $\cos\left(-\frac{15\pi}{2}\right) = \cos \frac{\pi}{2} = 0$

34. $\sin \frac{19\pi}{6} = \sin \frac{7\pi}{6} = -\frac{1}{2}$

35. $\sin\left(-\frac{9\pi}{4}\right) = \sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

36. $\cos\left(-\frac{8\pi}{3}\right) = \cos \frac{4\pi}{3} = -\frac{1}{2}$

37. $\sin t = \frac{1}{3}$

(a) $\sin(-t) = -\sin t = -\frac{1}{3}$

(b) $\csc(-t) = -\csc t = -3$

38. $\sin(-t) = \frac{3}{8}$

(a) $\sin t = -\sin(-t) = -\frac{3}{8}$

(b) $\csc t = \frac{1}{\sin t} = -\frac{8}{3}$

39. $\cos(-t) = -\frac{1}{5}$

(a) $\cos t = \cos(-t) = -\frac{1}{5}$

(b) $\sec(-t) = \frac{1}{\cos(-t)} = -5$

40. $\cos t = -\frac{3}{4}$

(a) $\cos(-t) = \cos t = -\frac{3}{4}$

(b) $\sec(-t) = \sec t = \frac{1}{\cos t} = -\frac{4}{3}$

41. $\sin t = \frac{4}{5}$

(a) $\sin(\pi - t) = \sin t = \frac{4}{5}$

(b) $\sin(t + \pi) = -\sin t = -\frac{4}{5}$

42. $\cos t = \frac{4}{5}$

(a) $\cos(\pi - t) = -\cos t = -\frac{4}{5}$

(b) $\cos(t + \pi) = -\cos t = -\frac{4}{5}$

43. $\sin \frac{\pi}{4} \approx 0.7071$

44. $\tan \frac{\pi}{3} \approx 1.7321$

45. $\csc 1.3 = \frac{1}{\sin 1.3} \approx 1.0378$

46. $\cot 1 = \frac{1}{\tan 1} \approx 0.6421$

47. $\cos(-1.7) \approx -0.1288$

48. $\cos(-2.5) \approx -0.8011$

49. $\csc 0.8 = \frac{1}{\sin 0.8} \approx 1.3940$

50. $\sec 1.8 = \frac{1}{0051.8} \approx -4.4014$

51. $\sec 22.8 = \frac{1}{\cos 22.8} \approx -1.4486$

52. $\sin(-0.9) \approx -0.7833$

53. (a) $\sin 5 = y \approx -1$
(b) $\cos 2 = x \approx -0.4$

54. (a) $\sin 0.75 = y \approx 0.7$
(b) $\cos 2.5 = x \approx -0.8$

55. (a) $\sin t = 0.25$

$t \approx 0.25$ or 2.89

(b) $\cos t = -0.25$

$t \approx 1.82$ or 4.46

56. (a) $\sin t = -0.75$

$t \approx 4.0$ or $t \approx 5.4$

(b) $\cos t = 0.75$

$t \approx 0.7$ or $t \approx 5.6$

57. $y(t) = \frac{1}{4}e^{-t}\cos 6t$

(a)

t	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
y	0.25	0.0138	-0.1501	-0.0249	0.0883

(b) From the table feature of a graphing utility we see that $y \approx 0$ when $t \approx 5$ seconds.

(c) As t increases, the displacement oscillates but decreases in amplitude.

58. $y(t) = \dfrac{1}{4}\cos 6t$

(a) $y(0) = \dfrac{1}{4}\cos 0 = 0.2500$ feet

(b) $y\left(\dfrac{1}{4}\right) = \dfrac{1}{4}\cos\dfrac{3}{2} \approx 0.0177$ feet

(c) $y\left(\dfrac{1}{2}\right) = \dfrac{1}{4}\cos 3 \approx -0.2475$ feet

59. False. $\sin(-t) = -\sin t$ means the function is odd, not that the sine of a negative angle is a negative number.

For example: $\sin\left(-\dfrac{3\pi}{2}\right) = -\sin\left(\dfrac{3\pi}{2}\right) = -(-1) = 1.$

Even though the angle is negative, the sine value is positive.

60. True. $\tan a = \tan(a - 6\pi)$ since the period of tan is π.

61. (a) The points have y-axis symmetry.

(b) $\sin t_1 = \sin(\pi - t_1)$ since they have the same y-value.

(c) $\cos(\pi - t_1) = -\cos t_1$ since the x-values have the opposite signs.

62. $\cos\theta = x = \cos(-\theta)$

$\sec\theta = \dfrac{1}{x} = \sec(-\theta)$

So $\sec\theta$ and $\cos\theta$ are even.

$\sin\theta = y$

$\sin(-\theta) = -y = -\sin\theta$

$\csc\theta = \dfrac{1}{y}$

$\csc(-\theta) = -\dfrac{1}{y} = -\csc\theta$

So $\sin\theta$ and $\csc\theta$ are odd.

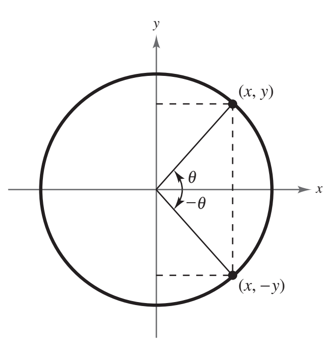
$\tan\theta = \dfrac{y}{x}$

$\tan(-\theta) = \dfrac{-y}{x} = -\tan\theta$

$\cot\theta = \dfrac{x}{y}$

$\cot(-\theta) = \dfrac{x}{-y} = -\cot\theta$

So $\tan\theta$ and $\cot\theta$ are odd.



63. $f(x) = \dfrac{1}{2}(3x - 2)$

$y = \dfrac{1}{2}(3x - 2)$

$x = \dfrac{1}{2}(3y - 2)$

$2x = 3y - 2$

$\dfrac{2}{3}x + \dfrac{2}{3} = y$

$f^{-1}(x) = \dfrac{2}{3}x + \dfrac{2}{3} = \dfrac{2}{3}(x + 1)$

64. $f(x) = \frac{1}{4}x^3 + 1$

$$y = \frac{1}{4}x^3 + 1$$

$$x = \frac{1}{4}y^3 + 1$$

$$x - 1 = \frac{1}{4}y^3$$

$$4(x - 1) = y^3$$

$$y = \sqrt[3]{4(x - 1)}$$

$$f^{-1}(x) = \sqrt[3]{4(x - 1)}$$

65. $f(x) = \sqrt{x^2 - 4}, x \ge 2$

$$y = \sqrt{x^2 - 4}$$

$$x = \sqrt{y^2 - 4}$$

$$x^2 = y^2 - 4$$

$$\pm\sqrt{x^2 + 4} = y$$

$$f^{-1}(x) = \sqrt{x^2 + 4}, x \ge 0$$

66. $f(x) = \frac{x + 2}{x - 4}$

$$y = \frac{x + 2}{x - 4}$$

$$x = \frac{y + 2}{y - 4}$$

$$x(y - 4) = y + 2$$

$$xy - 4x = y + 2$$

$$xy - y = 4x + 2$$

$$y(x - 1) = 4x + 2$$

$$y = \frac{2(2x + 1)}{x - 1}$$

$$f^{-1}(x) = \frac{2(2x + 1)}{x - 1}$$

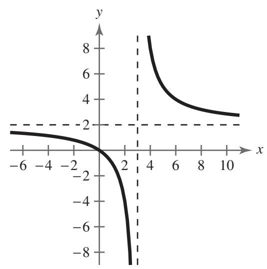
67. $f(x) = \frac{2x}{x - 3}$

Intercept: $(0, 0)$

Vertical asymptote: $x = 3$

Horizontal asymptote: $y = 2$

x	-1	0	1	2	4	5	6
y	$\frac{1}{2}$	0	-1	-4	8	5	4



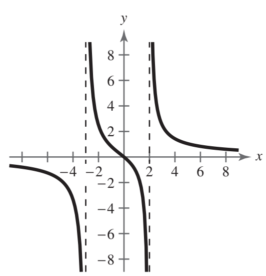
68. $f(x) = \frac{5x}{x^2 + x - 6} = \frac{5x}{(x + 3)(x - 2)}, x \ne -3, 2$

Horizontal asymptote: $x = 0$

Vertical asymptote: $x = -3, x = 2$

Intercept: $(0, 0)$

x	-6	-4	-2	0	1	3	5
y	$-\frac{5}{4}$	$-\frac{10}{3}$	$\frac{5}{2}$	0	$-\frac{5}{4}$	$\frac{5}{2}$	$\frac{25}{24}$



69. $f(x) = \frac{x^2 + 3x - 10}{2x^2 - 8} = \frac{(x + 5)(x - 2)}{2(x + 2)(x - 2)} = \frac{x + 5}{2(x + 2)}, x \ne 2$

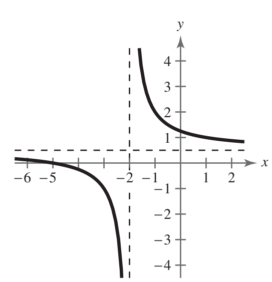
Intercepts: $(-5, 0), \left(0, \frac{5}{4}\right)$

Vertical asymptote: $x = -2$

Horizontal asymptote: $y = \frac{1}{2}$

Hole in the graph at $\left(2, \frac{7}{8}\right)$

x	-5	-4	-3	-1	0	1	3
y	0	$-\frac{1}{4}$	-1	2	$\frac{5}{4}$	1	$\frac{4}{5}$



70. $f(x) = \dfrac{x^3 - 6x^2 + x - 1}{2x^2 - 5x - 8} = \dfrac{1}{2}x - \dfrac{7}{4} - \dfrac{15(x + 4)}{4(2x^2 - 5x - 8)}$

x	-4	-3	$-\frac{3}{2}$	-1	0	2	3	4	7
y	$-\frac{15}{4}$	$-\frac{17}{5}$	$-\frac{155}{32}$	9	$\frac{1}{8}$	$\frac{3}{2}$	5	$-\frac{29}{4}$	1

Vertical asymptote: $2x^2 - 5x - 8 = 0$

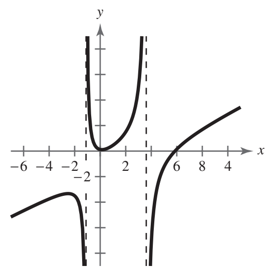
$$x = \frac{5 \pm \sqrt{(-5)^2 - (4)(2)(-8)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{89}}{4};\ x \approx -1.11,\ x \approx 3.61$$

Slant asymptote: $y = \dfrac{1}{2}x - \dfrac{7}{4}$

y-intercept: $\left(0, \dfrac{1}{8}\right)$

x-intercept: $(\approx 5.86, 0)$



Section 4.3 Right Triangle Trigonometry

- You should know the right triangle definition of trigonometric functions.

  (a) $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ (b) $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ (c) $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

  (d) $\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$ (e) $\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$ (f) $\cot\theta = \dfrac{\text{adj}}{\text{opp}}$

- You should know the following identities.

  (a) $\sin\theta = \dfrac{1}{\csc\theta}$ (b) $\csc\theta = \dfrac{1}{\sin\theta}$ (c) $\cos\theta = \dfrac{1}{\sec\theta}$

  (d) $\sec\theta = \dfrac{1}{\cos\theta}$ (e) $\tan\theta = \dfrac{1}{\cot\theta}$ (f) $\cot\theta = \dfrac{1}{\tan\theta}$

  (g) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ (h) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ (i) $\sin^2\theta + \cos^2\theta = 1$

  (j) $1 + \tan^2\theta = \sec^2\theta$ (k) $1 + \cot^2\theta = \csc^2\theta$

- You should know that two acute angles α and β are complementary if $\alpha + \beta = 90°$, and that cofunctions of complementary angles are equal.
- You should know the trigonometric function values of 30°, 45°, and 60°, or be able to construct triangles from which you can determine them.

Vocabulary Check

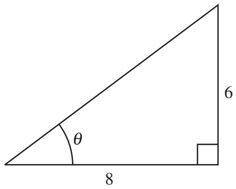
1. (i) $\dfrac{\text{hypotenuse}}{\text{adjacent}} = \sec\theta$ (v) (ii) $\dfrac{\text{adjacent}}{\text{opposite}} = \cot\theta$ (iv) (iii) $\dfrac{\text{hypotenuse}}{\text{opposite}} = \csc\theta$ (vi)

(iv) $\dfrac{\text{adjacent}}{\text{hypotenuse}} = \cos\theta$ (iii) (v) $\dfrac{\text{opposite}}{\text{hypotenuse}} = \sin\theta$ (i) (vi) $\dfrac{\text{opposite}}{\text{adjacent}} = \tan\theta$ (ii)

2. opposite; adjacent; hypotenuse

3. elevation; depression

1. hyp $= \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

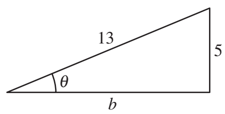


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{6}{10} = \frac{3}{5} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{10}{6} = \frac{5}{3}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{8}{10} = \frac{4}{5} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{10}{8} = \frac{5}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{6}{8} = \frac{3}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{8}{6} = \frac{4}{3}$$

2.



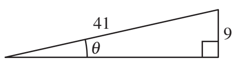
adj $= \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = 12$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{5}{13} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{13}{5}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{13} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{13}{12}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{5}{12} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{12}{5}$$

3. adj $= \sqrt{41^2 - 9^2} = \sqrt{1681 - 81} = \sqrt{1600} = 40$

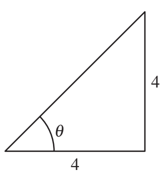


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{9}{41} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{41}{9}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{40}{41} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{41}{40}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{9}{40} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{40}{9}$$

4.



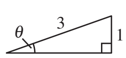
hyp $= \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{4\sqrt{2}}{4} = \sqrt{2}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{4\sqrt{2}}{4} = \sqrt{2}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{4} = 1 \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{4} = 1$$

5. adj $= \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$

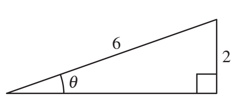


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{3} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = 3$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2\sqrt{2}}{3} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = 2\sqrt{2}$$

adj $= \sqrt{6^2 - 2^2} = \sqrt{32} = 4\sqrt{2}$



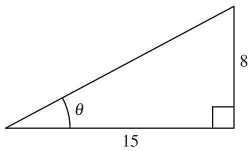
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2}{6} = \frac{1}{3} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{6}{2} = 3$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{6}{4\sqrt{2}} = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2}{4\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

The function values are the same since the triangles are similar and the corresponding sides are proportional.

6.

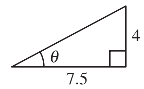


hyp $= \sqrt{15^2 + 8^2} = \sqrt{289} = 17$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{8}{17}$ $\quad \csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{17}{8}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{15}{17}$ $\quad \sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{17}{15}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{8}{15}$ $\quad \cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{15}{8}$



hyp $= \sqrt{7.5^2 + 4^2} = \dfrac{17}{2}$

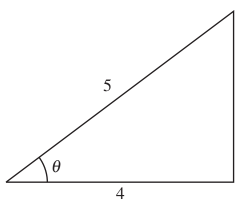
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{4}{17/2} = \dfrac{8}{17}$ $\quad \csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{17/2}{4} = \dfrac{17}{8}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{7.5}{17/2} = \dfrac{15}{17}$ $\quad \sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{17/2}{7.5} = \dfrac{17}{15}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{4}{7.5} = \dfrac{8}{15}$ $\quad \cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{7.5}{4} = \dfrac{15}{8}$

The function values are the same because the triangles are similar, and corresponding sides are proportional.

7. opp $= \sqrt{5^2 - 4^2} = 3$

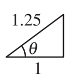


$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{3}{5}$ $\quad \csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{5}{3}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{4}{5}$ $\quad \sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{5}{4}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{3}{4}$ $\quad \cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{4}{3}$

opp $= \sqrt{1.25^2 - 1^2} = 0.75$



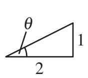
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{0.75}{1.25} = \dfrac{3}{5}$ $\quad \csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{1.25}{0.75} = \dfrac{5}{3}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{1}{1.25} = \dfrac{4}{5}$ $\quad \sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{1.25}{1} = \dfrac{5}{4}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{0.75}{1} = \dfrac{3}{4}$ $\quad \cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{1}{0.75} = \dfrac{4}{3}$

The function values are the same since the triangles are similar and the corresponding sides are proportional.

8.

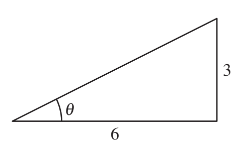


hyp $= \sqrt{1^2 + 2^2} = \sqrt{5}$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$ $\quad \csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{\sqrt{5}}{1} = \sqrt{5}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$ $\quad \sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{\sqrt{5}}{2}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{1}{2}$ $\quad \cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{2}{1} = 2$



hyp $= \sqrt{3^2 + 6^2} = 3\sqrt{5}$

$\sin\theta = \dfrac{3}{3\sqrt{5}} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$ $\quad \csc\theta = \dfrac{3\sqrt{5}}{3} = \sqrt{5}$

$\cos\theta = \dfrac{6}{3\sqrt{5}} = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$ $\quad \sec\theta = \dfrac{3\sqrt{5}}{6} = \dfrac{\sqrt{5}}{2}$

$\tan\theta = \dfrac{3}{6} = \dfrac{1}{2}$ $\quad \cot\theta = \dfrac{6}{3} = 2$

The function values are the same because the triangles are similar, and corresponding sides are proportional.

9. Given: $\sin\theta = \dfrac{3}{4} = \dfrac{\text{opp}}{\text{hyp}}$

$3^2 + (\text{adj})^2 = 4^2$

$\text{adj} = \sqrt{7}$

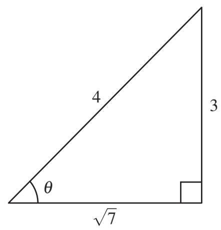
$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{\sqrt{7}}{4}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{3\sqrt{7}}{7}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{4}{3}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{4\sqrt{7}}{7}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{\sqrt{7}}{3}$



10. Given: $\cos\theta = \dfrac{5}{7} = \dfrac{\text{adj}}{\text{hyp}}$

$\text{opp} = \sqrt{7^2 - 5^2} = \sqrt{24} = 2\sqrt{6}$

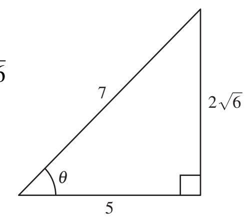
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{2\sqrt{6}}{7}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{2\sqrt{6}}{5}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{7}{2\sqrt{6}} = \dfrac{7\sqrt{6}}{12}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{7}{5}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{5}{2\sqrt{6}} = \dfrac{5\sqrt{6}}{12}$



11. Given: $\sec\theta = 2 = \dfrac{2}{1} = \dfrac{\text{hyp}}{\text{adj}}$

$(\text{opp})^2 + 1^2 = 2^2$

$\text{opp} = \sqrt{3}$

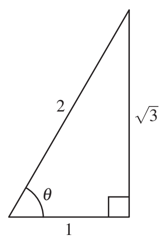
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{\sqrt{3}}{2}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{1}{2}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \sqrt{3}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{2\sqrt{3}}{3}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{\sqrt{3}}{3}$



12. Given: $\cot\theta = \dfrac{5}{1} = \dfrac{\text{adj}}{\text{opp}}$

$\text{hyp} = \sqrt{5^2 + 1^2} = \sqrt{26}$

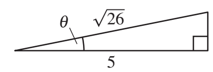
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{1}{\sqrt{26}} = \dfrac{\sqrt{26}}{26}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{5}{\sqrt{26}} = \dfrac{5\sqrt{26}}{26}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{1}{5}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{\sqrt{26}}{1} = \sqrt{26}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \dfrac{\sqrt{26}}{5}$



13. Given: $\tan\theta = 3 = \dfrac{3}{1} = \dfrac{\text{opp}}{\text{adj}}$

$3^2 + 1^2 = (\text{hyp})^2$

$\text{hyp} = \sqrt{10}$

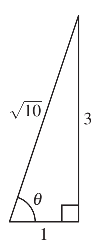
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{3\sqrt{10}}{10}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{\sqrt{10}}{10}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{\sqrt{10}}{3}$

$\sec\theta = \dfrac{\text{hyp}}{\text{adj}} = \sqrt{10}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{1}{3}$



14. Given: $\sec\theta = \dfrac{6}{1} = \dfrac{\text{hyp}}{\text{adj}}$

$\text{opp} = \sqrt{6^2 - 1^2} = \sqrt{35}$

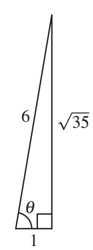
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{\sqrt{35}}{6}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{1}{6}$

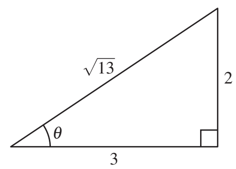
$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{\sqrt{35}}{1} = \sqrt{35}$

$\csc\theta = \dfrac{\text{hyp}}{\text{opp}} = \dfrac{6}{\sqrt{35}} = \dfrac{6\sqrt{35}}{35}$

$\cot\theta = \dfrac{\text{adj}}{\text{opp}} = \dfrac{1}{\sqrt{35}} = \dfrac{\sqrt{35}}{35}$

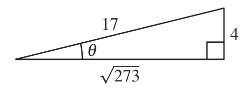


15. Given: $\cot\theta = \dfrac{3}{2} = \dfrac{\text{adj}}{\text{opp}}$



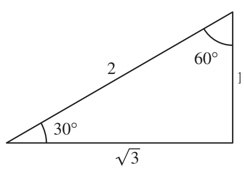
$$2^2 + 3^2 = (\text{hyp})^2$$
$$\text{hyp} = \sqrt{13}$$
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2}{3}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{13}}{2}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{13}}{3}$$

16. Given: $\csc\theta = \dfrac{17}{4} = \dfrac{\text{hyp}}{\text{opp}}$



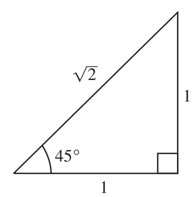
$$\text{adj} = \sqrt{17^2 - 4^2} = \sqrt{273}$$
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{17}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{273}}{17}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{\sqrt{273}} = \frac{4\sqrt{273}}{273}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{17}{\sqrt{273}} = \frac{17\sqrt{273}}{273}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{273}}{4}$$

17.



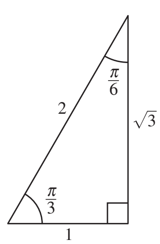
$$30° = 30°\left(\frac{\pi}{180°}\right) = \frac{\pi}{6}\text{ radian}$$
$$\sin 30° = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$$

18.



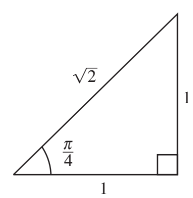
$$45° = 45°\left(\frac{\pi}{180°}\right) = \frac{\pi}{4}\text{ radian}$$
$$\cos 45° = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

19.



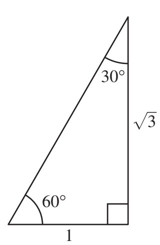
$$\frac{\pi}{3} = \frac{\pi}{3}\left(\frac{180°}{\pi}\right) = 60°$$
$$\tan\frac{\pi}{3} = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

20.



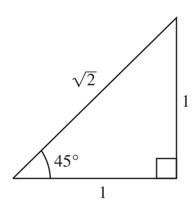
$$\frac{\pi}{4} = \frac{\pi}{4}\left(\frac{180°}{\pi}\right) = 45°$$
$$\sec\frac{\pi}{4} = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

21.



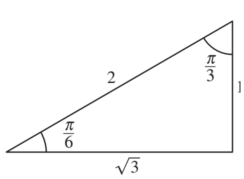
$$\cot\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = \frac{\text{adj}}{\text{opp}}$$
$$\theta = 60° = \frac{\pi}{3}\text{ radian}$$

22.



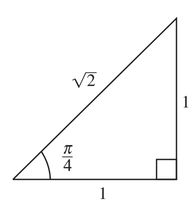
$$\csc\theta = \sqrt{2} = \frac{\text{hyp}}{\text{opp}}$$
$$\theta = 45° = 45°\left(\frac{\pi}{180°}\right) = \frac{\pi}{4}\text{ radian}$$

23.



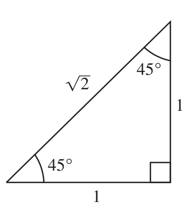
$$\frac{\pi}{6} = \frac{\pi}{6}\left(\frac{180°}{\pi}\right) = 30°$$
$$\cos\frac{\pi}{6} = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

24.



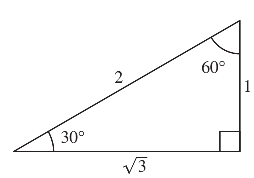
$$\theta = \frac{\pi}{4} = \frac{\pi}{4}\left(\frac{180°}{\pi}\right) = 45°$$
$$\sin\frac{\pi}{4} = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

25.



$$\cot\theta = 1 = \frac{1}{1} = \frac{\text{adj}}{\text{opp}}$$
$$\theta = 45° = 45°\left(\frac{\pi}{180°}\right)$$
$$= \frac{\pi}{4}\text{ radian}$$

26.



$$\tan\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = \frac{\text{opp}}{\text{adj}}$$
$$\theta = 30° = 30°\left(\frac{\pi}{180°}\right)$$
$$= \frac{\pi}{6}\text{ radian}$$

27. $\sin 60° = \dfrac{\sqrt{3}}{2},\ \cos 60° = \dfrac{1}{2}$

(a) $\tan 60° = \dfrac{\sin 60°}{\cos 60°} = \sqrt{3}$

(b) $\sin 30° = \cos 60° = \dfrac{1}{2}$

(c) $\cos 30° = \sin 60° = \dfrac{\sqrt{3}}{2}$

(d) $\cot 60° = \dfrac{\cos 60°}{\sin 60°} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

28. $\sin 30° = \dfrac{1}{2},\ \tan 30° = \dfrac{\sqrt{3}}{3}$

(a) $\csc 30° = \dfrac{1}{\sin 30°} = 2$

(b) $\cot 60° = \tan(90° - 60°) = \tan 30° = \dfrac{\sqrt{3}}{3}$

(c) $\cos 30° = \dfrac{\sin 30°}{\tan 30°} = \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{3}} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

(d) $\cot 30° = \dfrac{1}{\tan 30°} = \dfrac{3}{\sqrt{3}} = \dfrac{3\sqrt{3}}{3} = \sqrt{3}$

29. $\csc\theta = \dfrac{\sqrt{13}}{2},\ \sec\theta = \dfrac{\sqrt{13}}{3}$

(a) $\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{2}{\sqrt{13}} = \dfrac{2\sqrt{13}}{13}$

(b) $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{3}{\sqrt{13}} = \dfrac{3\sqrt{13}}{13}$

(c) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{2\sqrt{13}}{13}}{\frac{3\sqrt{13}}{13}} = \dfrac{2}{3}$

(d) $\sec(90° - \theta) = \csc\theta = \dfrac{\sqrt{13}}{2}$

30. $\sec\theta = 5,\ \tan\theta = 2\sqrt{6}$

(a) $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{5}$

(b) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{2\sqrt{6}} = \dfrac{\sqrt{6}}{12}$

(c) $\cot(90° - \theta) = \tan\theta = 2\sqrt{6}$

(d) $\sin\theta = \tan\theta\cos\theta = \left(2\sqrt{6}\right)\left(\dfrac{1}{5}\right) = \dfrac{2\sqrt{6}}{5}$

31. $\cos\alpha = \dfrac{1}{3}$

(a) $\sec\alpha = \dfrac{1}{\cos\alpha} = 3$

(b) $\sin^2\alpha + \cos^2\alpha = 1$

$$\sin^2\alpha + \left(\frac{1}{3}\right)^2 = 1$$

$$\sin^2\alpha = \frac{8}{9}$$

$$\sin\alpha = \frac{2\sqrt{2}}{3}$$

(c) $\cot\alpha = \dfrac{\cos\alpha}{\sin\alpha} = \dfrac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$

(d) $\sin(90° - \alpha) = \cos\alpha = \dfrac{1}{3}$

32. $\tan\beta = 5$

(a) $\cot\beta = \dfrac{1}{\tan\beta} = \dfrac{1}{5}$

(b) $\cos\beta = \dfrac{1}{\sec\beta} = \dfrac{1}{\sqrt{1 + \tan^2\beta}}$

$$= \frac{1}{\sqrt{1 + 5^2}} = \frac{1}{\sqrt{26}} = \frac{\sqrt{26}}{26}$$

(c) $\tan(90° - \beta) = \cot\beta = \dfrac{1}{\tan\beta} = \dfrac{1}{5}$

(d) $\csc\beta = \sqrt{1 + \cot^2\beta}$

$$= \sqrt{1 + \left(\frac{1}{5}\right)^2}$$

$$= \sqrt{1 + \frac{1}{25}} = \sqrt{\frac{26}{25}} = \frac{\sqrt{26}}{5}$$

33. $\tan\theta\cot\theta = \tan\theta\left(\dfrac{1}{\tan\theta}\right) = 1$

34. $\cos\theta\sec\theta = \cos\theta\left(\dfrac{1}{\cos\theta}\right) = 1$

35. $\tan \alpha \cos \alpha = \left(\dfrac{\sin \alpha}{\cos \alpha}\right) \cos \alpha = \sin \alpha$

36. $\cot \alpha \sin \alpha = \dfrac{\cos \alpha}{\sin \alpha} \sin \alpha = \cos \alpha$

37. $$\begin{aligned}(1 + \cos \theta)(1 - \cos \theta) &= 1 - \cos^2 \theta \\ &= (\sin^2 \theta + \cos^2 \theta) - \cos^2 \theta \\ &= \sin^2 \theta\end{aligned}$$

38. $(1 + \sin \theta)(1 - \sin \theta) = 1 - \sin^2 \theta = \cos^2 \theta$

39. $$\begin{aligned}(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) &= \sec^2 \theta - \tan^2 \theta \\ &= (1 + \tan^2 \theta) - \tan^2 \theta \\ &= 1\end{aligned}$$

40. $$\begin{aligned}\sin^2 \theta - \cos^2 \theta &= \sin^2 \theta - (1 - \sin^2 \theta) \\ &= \sin^2 \theta - 1 + \sin^2 \theta \\ &= 2 \sin^2 \theta - 1\end{aligned}$$

41. $$\begin{aligned}\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \\ &= \frac{1}{\sin \theta \cos \theta} \\ &= \frac{1}{\sin \theta} \cdot \frac{1}{\cos \theta} \\ &= \csc \theta \sec \theta\end{aligned}$$

42. $$\begin{aligned}\frac{\tan \beta + \cot \beta}{\tan \beta} &= \frac{\tan \beta}{\tan \beta} + \frac{\cot \beta}{\tan \beta} \\ &= 1 + \frac{\cot \beta}{\left(\dfrac{1}{\cot \beta}\right)} \\ &= 1 + \cot^2 \beta = \csc^2 \beta\end{aligned}$$

43. (a) $\sin 10° \approx 0.1736$

(b) $\cos 80° \approx 0.1736$

Note: $\cos 80° = \sin(90° - 80°) = \sin 10°$

44. (a) $\tan 23.5° \approx 0.4348$

(b) $\cot 66.5° = \dfrac{1}{\tan 66.5°} \approx 0.4348$

45. (a) $\sin 16.35° \approx 0.2815$

(b) $\csc 16.35° = \dfrac{1}{\sin 16.35°} \approx 3.5523$

46. (a) $\cos 16° \, 18' = \cos\left(16 + \dfrac{18}{60}\right)° \approx 0.9598$

(b) $\sin 73° \, 56' = \sin\left(73 + \dfrac{56}{60}\right)° \approx 0.9609$

47. (a) $\sec 42° \, 12' = \sec 42.2° = \dfrac{1}{\cos 42.2°} \approx 1.3499$

(b) $\csc 48° \, 7' = \dfrac{1}{\sin\left(48 + \frac{7}{60}\right)°} \approx 1.3432$

48. (a) $$\begin{aligned}\cos 4° \, 50' \, 15'' &= \cos\left(4 + \frac{50}{60} + \frac{15}{3600}\right)° \\ &\approx 0.9964\end{aligned}$$

(b) $$\begin{aligned}\sec 4° \, 50' \, 15'' &= \frac{1}{\cos 4° \, 50' \, 15''} \\ &\approx 1.0036\end{aligned}$$

49. (a) $\cot 11° \, 15' = \dfrac{1}{\tan 11.25°} \approx 5.0273$

(b) $\tan 11° \, 15' = \tan 11.25° \approx 0.1989$

50. (a) $\sec 56° \, 8' \, 10'' = \sec\left(56 + \dfrac{8}{60} = \dfrac{10}{3600}\right)° \approx 1.7946$

(b) $\cos 56° \, 8' \, 10'' = \cos\left(56 + \dfrac{8}{60} + \dfrac{10}{3600}\right)° \approx 0.5572$

51. (a) $\csc 32° \, 40' \, 3'' = \dfrac{1}{\sin 32.6675°} \approx 1.8527$

(b) $\tan 44° \, 28' \, 16'' \approx \tan 44.4711° \approx 0.9817$

52. (a) $\sec\left(\dfrac{9}{5} \cdot 20 + 32\right)° \approx 2.6695$

(b) $\cot\left(\dfrac{9}{5} \cdot 30 + 32\right)° \approx 0.0699$

53. (a) $\sin \theta = \dfrac{1}{2} \implies \theta = 30° = \dfrac{\pi}{6}$

(b) $\csc \theta = 2 \implies \theta = 30° = \dfrac{\pi}{6}$

54. (a) $\cos \theta = \dfrac{\sqrt{2}}{2} \implies \theta = 45° = \dfrac{\pi}{4}$

(b) $\tan \theta = 1 \implies \theta = 45° = \dfrac{\pi}{4}$

55. (a) $\sec \theta = 2 \implies \theta = 60° = \dfrac{\pi}{3}$

(b) $\cot \theta = 1 \implies \theta = 45° = \dfrac{\pi}{4}$

56. (a) $\tan \theta = \sqrt{3} \implies \theta = 60° = \dfrac{\pi}{3}$

(b) $\cos \theta = \dfrac{1}{2} \implies \theta = 60° = \dfrac{\pi}{3}$

57. (a)

$\csc \theta = \dfrac{2\sqrt{3}}{3} \implies \theta = 60° = \dfrac{\pi}{3}$

(b) $\sin \theta = \dfrac{\sqrt{2}}{2} \implies \theta = 45° = \dfrac{\pi}{4}$

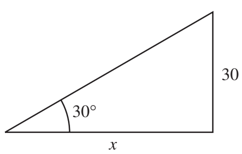
58. (a) $\cot \theta = \dfrac{\sqrt{3}}{3}$

$\tan \theta = \dfrac{3}{\sqrt{3}} = \sqrt{3} \implies \theta = 60° = \dfrac{\pi}{3}$

(b) $\sec \theta = \sqrt{2}$

$\cos \theta = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} \implies \theta = 45° = \dfrac{\pi}{4}$

59.

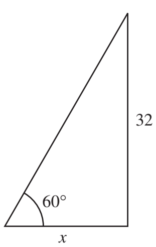


$$\tan 30° = \frac{30}{x}$$
$$\frac{1}{\sqrt{3}} = \frac{30}{x}$$
$$x = 30\sqrt{3}$$

60. $\sin 60° = \dfrac{y}{18}$

$$y = 18 \sin 60° = 18\left(\frac{\sqrt{3}}{2}\right) = 9\sqrt{3}$$

61.



$$\tan 60° = \frac{32}{x}$$
$$\sqrt{3} = \frac{32}{x}$$
$$\sqrt{3}x = 32$$
$$x = \frac{32}{\sqrt{3}} = \frac{32\sqrt{3}}{3}$$

62. $\sin 45° = \dfrac{20}{r}$

$$r = \frac{20}{\sin 45°} = \frac{20}{\sqrt{2}/2} = 20\sqrt{2}$$

63. $\tan 82° = \dfrac{x}{45}$

$x = 45 \tan 82°$

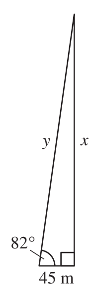
Height of the building:

$123 + 45 \tan 82° \approx 443.2$ meters

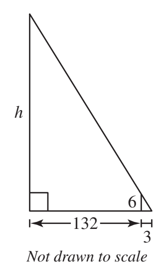
Distance between friends:

$\cos 82° = \dfrac{45}{y} \implies y = \dfrac{45}{\cos 82°}$

≈ 323.34 meters



64. (a)

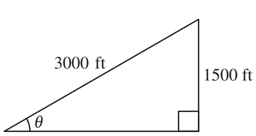


(b) $\tan \theta = \dfrac{6}{3} = \dfrac{h}{135}$

(c) $2(135) = h$

$h = 270$ feet

65.



$\sin \theta = \dfrac{1500}{3000} = \dfrac{1}{2}$

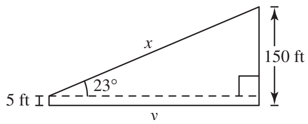
$\theta = 0° = \dfrac{\pi}{6}$

66. $\tan \theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan 54° = \dfrac{w}{100}$

$w = 100 \tan 54° \approx 137.6$ feet

67.



(a) $\sin 23° = \dfrac{145}{x}$

$$x = \frac{145}{\sin 23°} \approx 371.1 \text{ feet}$$

(b) $\tan 23° = \dfrac{145}{y}$

$$y = \frac{145}{\tan 23°} \approx 341.6 \text{ feet}$$

(c) Moving down the line:

$$\frac{145/\sin 23°}{6} \approx 61.85 \text{ feet per second}$$

Dropping vertically:

$$\frac{145}{6} \approx 24.17 \text{ feet per second}$$

68. Let h = the height of the mountain.

Let x = the horizontal distance from where the 9° angle of elevation is sighted to the point at that level directly below the mountain peak.

Then $\tan 3.5° = \dfrac{h}{x + 13}$ and $\tan 9° = \dfrac{h}{x}$.

$$\tan 9° = \frac{h}{x} \Rightarrow x = \frac{h}{\tan 9°}$$

Substitute $x = \dfrac{h}{\tan 9°}$ into the expression for $\tan 3.5°$.

$$\tan 3.5° = \frac{h}{\dfrac{h}{\tan 9°} + 13}$$

$$\tan 3.5° = \frac{h \tan 9°}{h + 13 \tan 9°}$$

$$h \tan 3.5° + 13 \tan 9° \tan 3.5° = h \tan 9°$$

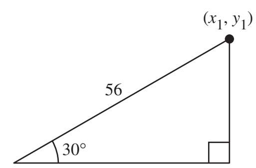
$$13 \tan 9° \tan 3.5° = h(\tan 9° - \tan 3.5°)$$

$$\frac{13 \tan 9° \tan 3.5°}{\tan 9° - \tan 3.5°} = h$$

$$1.2953 \approx h$$

The mountain is about 1.3 miles high.

69.



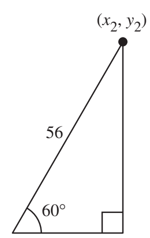
$$\sin 30° = \frac{y_1}{56}$$

$$y_1 = (\sin 30°)(56) = \left(\frac{1}{2}\right)(56) = 28$$

$$\cos 30° = \frac{x_1}{56}$$

$$x_1 = \cos 30°(56) = \frac{\sqrt{3}}{2}(56) = 28\sqrt{3}$$

$$(x_1, y_1) = \left(28\sqrt{3}, 28\right)$$



$$\sin 60° = \frac{y_2}{56}$$

$$y_2 = \sin 60°(56) = \left(\frac{\sqrt{3}}{2}\right)(56) = 28\sqrt{3}$$

$$\cos 60° = \frac{x_2}{56}$$

$$x_2 = (\cos 60°)(56) = \left(\frac{1}{2}\right)(56) = 28$$

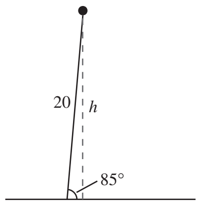
$$(x_2, y_2) = \left(28, 28\sqrt{3}\right)$$

70. $\tan 3° = \dfrac{x}{15}$

$$x = 15 \tan 3°$$

$$d = 5 + 2x = 5 + 2(15 \tan 3°) \approx 6.57 \text{ centimeters}$$

71. (a)



(b) $\sin 85° = \dfrac{h}{20}$

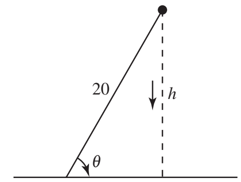
(c) $h = 20 \sin 85° \approx 19.9$ meters

(d) The side of the triangle labeled h will become shorter.

(e)

Angle, θ	Height (in meters)
80°	19.7
70°	18.8
60°	17.3
50°	15.3
40°	12.9
30°	10.0
20°	6.8
10°	3.5

(f) The height of the balloon decreases as θ decreases.



72. $x \approx 9.4, \; y \approx 3.4$

$\sin 20° = \dfrac{y}{10} \approx 0.34 \qquad \cot 20° = \dfrac{x}{y} \approx 2.75$

$\cos 20° = \dfrac{x}{10} \approx 0.94 \qquad \sec 20° = \dfrac{10}{x} \approx 1.06$

$\tan 20° = \dfrac{y}{x} \approx 0.36 \qquad \csc 20° = \dfrac{10}{y} \approx 2.92$

73. True,

$\csc x = \dfrac{1}{\sin x} \implies \sin 60° \csc 60° = \sin 60°\left(\dfrac{1}{\sin 60°}\right) = 1$

74. True, $\sec 30° = \csc 60°$ because $\sec(90° - \theta) = \csc \theta$.

75. False, $\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} = \sqrt{2} \neq 1$

76. True, $\cot^2 10° - \csc^2 10° = -1$ because

$$1 + \cot^2 \theta = \csc^2 \theta$$
$$\cot^2 \theta = \csc^2 \theta - 1$$
$$\cot^2 \theta - \csc^2 \theta = -1.$$

77. False, $\dfrac{\sin 60°}{\sin 30°} = \dfrac{\cos 30°}{\sin 30°} = \cot 30° \approx 1.7321;$

$\sin 2° \approx 0.0349$

78. False, $\tan[(5°)^2] \neq \tan^2(5°)$.

$\tan[(5°)^2] = \tan 25° \approx 0.4663$

$\tan^2(5°) = (\tan 5°)(\tan 5°) \approx 0.0077$

79. This is true because the corresponding sides of similar triangles are proportional.

80. Yes. Given $\tan \theta$, $\sec \theta$ can be found from the identity $1 + \tan^2 \theta = \sec^2 \theta$.

81. (a)

θ	0.1	0.2	0.3	0.4	0.5
$\sin \theta$	0.0998	0.1987	0.2955	0.3894	0.4794

(b) In the interval $(0, 0.5]$, $\theta > \sin \theta$.

(c) As θ approaches 0, $\sin \theta$ approaches θ.

82. (a)

θ	0°	18°	36°	54°	72°	90°
$\sin \theta$	0	0.3090	0.5878	0.8090	0.9511	1
$\cos \theta$	1	0.9511	0.8090	0.5878	0.3090	0

—CONTINUED—

82. —CONTINUED—

(b) $\sin\theta$ increases from 0 to 1 as θ increases from 0° to 90°.

(c) $\cos\theta$ decreases from 1 to 0 as θ increases from 0° to 90°.

(d) As the angle increases the length of the side opposite the angle increases relative to the length of the hypotenuse and the length of the side adjacent to the angle decreases relative to the length of the hypotenuse. Thus the sine increases and the cosine decreases.

83. $$\frac{x^2-6x}{x^2+4x-12}\cdot\frac{x^2+12x+36}{x^2-36}=\frac{x\cancel{(x-6)}}{\cancel{(x+6)}(x-2)}\cdot\frac{\cancel{(x+6)}\cancel{(x+6)}}{\cancel{(x+6)}\cancel{(x-6)}}$$
$$=\frac{x}{x-2},\ x\neq\pm 6$$

84. $$\frac{2t^2+5t-12}{9-4t^2}\div\frac{t^2-16}{4t^2+12t+9}=\frac{2t^2+5t-12}{9-4t^2}\cdot\frac{4t^2+12t+9}{t^2-16}$$
$$=\frac{(2t-3)(t+4)}{(3+2t)(3-2t)}\cdot\frac{(2t+3)(2t+3)}{(t+4)(t-4)}=-\frac{(2t+3)}{(t-4)}=\frac{2t+3}{4-t},\ t\neq\pm\frac{3}{2},\ -4$$

85. $$\frac{3}{x+2}-\frac{2}{x-2}+\frac{x}{x^2+4x+4}=\frac{3(x+2)(x-2)-2(x+2)^2+x(x-2)}{(x-2)(x+2)^2}$$
$$=\frac{3(x^2-4)-2(x^2+4x+4)+x^2-2x}{(x-2)(x+2)^2}$$
$$=\frac{2x^2-10x-20}{(x-2)(x+2)^2}=\frac{2(x^2-5x-10)}{(x-2)(x+2)^2}$$

86. $$\frac{\left(\frac{3}{x}-\frac{1}{4}\right)}{\left(\frac{12}{x}-1\right)}=\frac{\frac{12-x}{4x}}{\frac{12-x}{x}}=\frac{12-x}{4x}\cdot\frac{x}{12-x}=\frac{1}{4},\ x\neq 0,\ 12$$

Section 4.4 Trigonometric Functions of Any Angle

- Know the Definitions of Trigonometric Functions of Any Angle.
  If θ is in standard position, (x, y) a point on the terminal side and $r=\sqrt{x^2+y^2}\neq 0$, then:

  $\sin\theta=\frac{y}{r}$ $\qquad$ $\csc\theta=\frac{r}{y},\ y\neq 0$

  $\cos\theta=\frac{x}{r}$ $\qquad$ $\sec\theta=\frac{r}{x},\ x\neq 0$

  $\tan\theta=\frac{y}{x},\ x\neq 0$ $\qquad$ $\cot\theta=\frac{x}{y},\ y\neq 0$
- You should know the signs of the trigonometric functions in each quadrant.
- You should know the trigonometric function values of the quadrant angles 0, $\frac{\pi}{2}$, π, and $\frac{3\pi}{2}$.
- You should be able to find reference angles.
- You should be able to evaluate trigonometric functions of any angle. (Use reference angles.)
- You should know that the period of sine and cosine is 2π.

Vocabulary Check

1. $\sin\theta = \frac{y}{r}$ **2.** $\csc\theta$ **3.** $\tan\theta = \frac{y}{x}$

4. $\frac{r}{x}$ **5.** $\frac{x}{r} = \cos\theta$ **6.** $\cot\theta$

7. reference

1. (a) $(x, y) = (4, 3)$

$r = \sqrt{16 + 9} = 5$

$\sin\theta = \frac{y}{r} = \frac{3}{5}$ $\csc\theta = \frac{r}{y} = \frac{5}{3}$

$\cos\theta = \frac{x}{r} = \frac{4}{5}$ $\sec\theta = \frac{r}{x} = \frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{3}{4}$ $\cot\theta = \frac{x}{y} = \frac{4}{3}$

(b) $(x, y) = (8, -15)$

$r = \sqrt{64 + 225} = 17$

$\sin\theta = \frac{y}{r} = -\frac{15}{17}$ $\csc\theta = \frac{r}{y} = -\frac{17}{15}$

$\cos\theta = \frac{x}{r} = \frac{8}{17}$ $\sec\theta = \frac{r}{x} = \frac{17}{8}$

$\tan\theta = \frac{y}{x} = -\frac{15}{8}$ $\cot\theta = \frac{x}{y} = -\frac{8}{15}$

2. (a) $x = -12, y = -5$

$r = \sqrt{(-12)^2 + (-5)^2} = 13$

$\sin\theta = \frac{y}{r} = -\frac{5}{13}$

$\cos\theta = \frac{x}{r} = -\frac{12}{13}$

$\tan\theta = \frac{y}{x} = \frac{-5}{-12} = \frac{5}{12}$

$\csc\theta = \frac{r}{y} = \frac{13}{-5} = \frac{13}{-5}$

$\sec\theta = \frac{r}{x} = \frac{13}{-12} = -\frac{13}{12}$

$\cot\theta = \frac{x}{y} = \frac{-12}{-5} = \frac{12}{5}$

(b) $x = -1, y = 1$

$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$\sin\theta = \frac{y}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\tan\theta = \frac{y}{x} = \frac{1}{-1} = -1$

$\csc\theta = \frac{r}{y} = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{2}}{-1} = -\sqrt{2}$

$\cot\theta = \frac{x}{y} = \frac{-1}{1} = -1$

3. (a) $(x, y) = \left(-\sqrt{3}, -1\right)$

$r = \sqrt{3 + 1} = 2$

$\sin\theta = \frac{y}{r} = -\frac{1}{2}$ $\csc\theta = \frac{r}{y} = -2$

$\cos\theta = \frac{x}{r} = -\frac{\sqrt{3}}{2}$ $\sec\theta = \frac{r}{x} = -\frac{2\sqrt{3}}{3}$

$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{3}$ $\cot\theta = \frac{x}{y} = \sqrt{3}$

(b) $(x, y) = (-4, 1)$

$r = \sqrt{16 + 1} = \sqrt{17}$

$\sin\theta = \frac{y}{r} = \frac{\sqrt{17}}{17}$ $\csc\theta = \frac{r}{y} = \sqrt{17}$

$\cos\theta = \frac{x}{r} = -\frac{4\sqrt{17}}{17}$ $\sec\theta = \frac{r}{x} = -\frac{\sqrt{17}}{4}$

$\tan\theta = \frac{y}{x} = -\frac{1}{4}$ $\cot\theta = \frac{x}{y} = -4$

4. (a) $x = 3, y = 1$

$r = \sqrt{3^2 + 1^2} = \sqrt{10}$

$\sin\theta = \dfrac{y}{r} = \dfrac{1}{\sqrt{10}} = \dfrac{\sqrt{10}}{10}$

$\cos\theta = \dfrac{x}{r} = \dfrac{3}{\sqrt{10}} = \dfrac{3\sqrt{10}}{10}$

$\tan\theta = \dfrac{y}{x} = \dfrac{1}{3}$

$\csc\theta = \dfrac{r}{y} = \dfrac{\sqrt{10}}{1} = \sqrt{10}$

$\sec\theta = \dfrac{r}{x} = \dfrac{\sqrt{10}}{3}$

$\cot\theta = \dfrac{x}{y} = \dfrac{3}{1} = 3$

(b) $x = 4, y = -4$

$r = \sqrt{4^2 + (-4)^2} = 4\sqrt{2}$

$\sin\theta = \dfrac{y}{r} = \dfrac{-4}{4\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$

$\cos\theta = \dfrac{x}{r} = \dfrac{4}{4\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\tan\theta = \dfrac{y}{x} = \dfrac{-4}{4} = -1$

$\csc\theta = \dfrac{r}{y} = \dfrac{4\sqrt{2}}{-4} = -\sqrt{2}$

$\sec\theta = \dfrac{r}{x} = \dfrac{4\sqrt{2}}{4} = \sqrt{2}$

$\cot\theta = \dfrac{x}{y} = \dfrac{4}{-4} = -1$

5. $(x, y) = (7, 24)$

$r = \sqrt{49 + 576} = 25$

$\sin\theta = \dfrac{y}{r} = \dfrac{24}{25}$

$\cos\theta = \dfrac{x}{r} = \dfrac{7}{25}$

$\tan\theta = \dfrac{y}{x} = \dfrac{24}{7}$

$\csc\theta = \dfrac{r}{y} = \dfrac{25}{24}$

$\sec\theta = \dfrac{r}{x} = \dfrac{25}{7}$

$\cot\theta = \dfrac{x}{y} = \dfrac{7}{24}$

6. $x = 8, y = 15$

$r = \sqrt{8^2 + 15^2} = 17$

$\sin\theta = \dfrac{y}{r} = \dfrac{15}{17}$

$\cos\theta = \dfrac{x}{r} = \dfrac{8}{17}$

$\tan\theta = \dfrac{y}{x} = \dfrac{15}{8}$

$\csc\theta = \dfrac{r}{y} = \dfrac{17}{15}$

$\sec\theta = \dfrac{r}{x} = \dfrac{17}{8}$

$\cot\theta = \dfrac{x}{y} = \dfrac{8}{15}$

7. $(x, y) = (-4, 10)$

$r = \sqrt{16 + 100} = 2\sqrt{29}$

$\sin\theta = \dfrac{y}{r} = \dfrac{5\sqrt{29}}{29}$

$\cos\theta = \dfrac{x}{r} = -\dfrac{2\sqrt{29}}{29}$

$\tan\theta = \dfrac{y}{x} = -\dfrac{5}{2}$

$\csc\theta = \dfrac{r}{y} = \dfrac{\sqrt{29}}{5}$

$\sec\theta = \dfrac{r}{x} = -\dfrac{\sqrt{29}}{2}$

$\cot\theta = \dfrac{x}{y} = -\dfrac{2}{5}$

8. $x = -5, y = -2$

$r = \sqrt{(-5)^2 + (-2)^2} = \sqrt{29}$

$\sin\theta = \dfrac{y}{r} = \dfrac{-2}{\sqrt{29}} = -\dfrac{2\sqrt{29}}{29}$

$\cos\theta = \dfrac{x}{r} = \dfrac{-5}{\sqrt{29}} = -\dfrac{5\sqrt{29}}{29}$

$\tan\theta = \dfrac{y}{x} = \dfrac{-2}{-5} = \dfrac{2}{5}$

$\csc\theta = \dfrac{r}{y} = \dfrac{\sqrt{29}}{-2} = -\dfrac{\sqrt{29}}{2}$

$\sec\theta = \dfrac{r}{x} = \dfrac{\sqrt{29}}{-5} = -\dfrac{\sqrt{29}}{5}$

$\cot\theta = \dfrac{x}{y} = \dfrac{-5}{-2} = \dfrac{5}{2}$

9. $(x, y) = (-3.5, 6.8)$

$r = \sqrt{12.25 + 46.24} = \dfrac{\sqrt{5849}}{10}$

$\sin\theta = \dfrac{y}{r} = \dfrac{68\sqrt{5849}}{5849} \approx 0.9$

$\cos\theta = \dfrac{x}{r} = -\dfrac{35\sqrt{5849}}{5849} \approx -0.5$

$\tan\theta = \dfrac{y}{x} = -\dfrac{68}{35} \approx -1.9$

$\csc\theta = \dfrac{r}{y} = \dfrac{\sqrt{5849}}{68} \approx 1.1$

$\sec\theta = \dfrac{r}{x} = -\dfrac{\sqrt{5849}}{35} \approx -2.2$

$\cot\theta = \dfrac{x}{y} = -\dfrac{35}{68} \approx -0.5$

10. $x = 3\frac{1}{2} = \frac{7}{2}, y = -7\frac{3}{4} = -\frac{31}{4}$

$r = \sqrt{\left(\frac{7}{2}\right)^2 + \left(-\frac{31}{4}\right)^2} = \frac{\sqrt{1157}}{4}$

$\sin\theta = \frac{y}{r} = \frac{-31/4}{\sqrt{1157}/4} = -\frac{31\sqrt{1157}}{1157} \approx -0.9$ $\qquad$ $\csc\theta = \frac{r}{y} = \frac{\sqrt{1157}/4}{-31/4} = -\frac{\sqrt{1157}}{31} \approx -1.1$

$\cos\theta = \frac{x}{r} = \frac{7/2}{\sqrt{1157}/4} = \frac{14\sqrt{1157}}{1157} \approx 0.4$ $\qquad$ $\sec\theta = \frac{r}{x} = \frac{\sqrt{1157}/4}{7/2} = \frac{\sqrt{1157}}{14} \approx 2.4$

$\tan\theta = \frac{y}{x} = \frac{-31/4}{7/2} = -\frac{31}{14} \approx -2.2$ $\qquad$ $\cot\theta = \frac{x}{y} = \frac{7/2}{-31/4} = -\frac{14}{31} \approx -0.5$

11. $\sin\theta < 0 \implies \theta$ lies in Quadrant III or in Quadrant IV.

$\cos\theta < 0 \implies \theta$ lies in Quadrant II or in Quadrant III.

$\sin\theta < 0$ *and* $\cos\theta < 0 \implies \theta$ lies in Quadrant III.

12. $\sin\theta > 0$ and $\cos\theta > 0$

$\frac{y}{r} > 0$ and $\frac{x}{r} > 0$

Quadrant I

13. $\sin\theta > 0 \implies \theta$ lies in Quadrant I or in Quadrant II.

$\tan\theta < 0 \implies \theta$ lies in Quadrant II or in Quadrant IV.

$\sin\theta > 0$ *and* $\tan\theta < 0 \implies \theta$ lies in Quadrant II.

14. $\sec\theta > 0$ and $\cot\theta < 0$

$\frac{r}{x} > 0$ and $\frac{x}{y} < 0$

Quadrant IV

15. $\sin\theta = \frac{y}{r} = \frac{3}{5} \implies x^2 = 25 - 9 = 16$

θ in Quadrant II $\implies x = -4$

$\sin\theta = \frac{y}{r} = \frac{3}{5}$ $\qquad$ $\csc\theta = \frac{r}{y} = \frac{5}{3}$

$\cos\theta = \frac{x}{r} = -\frac{4}{5}$ $\qquad$ $\sec\theta = \frac{r}{x} = -\frac{5}{4}$

$\tan\theta = \frac{y}{x} = -\frac{3}{4}$ $\qquad$ $\cot\theta = \frac{x}{y} = -\frac{4}{3}$

16. $\cos\theta = \frac{x}{r} = \frac{-4}{5} \implies y^2 = 25 - 16 = 9$

θ in Quadrant III $\implies y = -3$

$\sin\theta = \frac{y}{r} = -\frac{3}{5}$ $\qquad$ $\csc\theta = -\frac{5}{3}$

$\cos\theta = \frac{x}{r} = -\frac{4}{5}$ $\qquad$ $\sec\theta = -\frac{5}{4}$

$\tan\theta = \frac{y}{x} = \frac{3}{4}$ $\qquad$ $\cot\theta = \frac{4}{3}$

17. $\tan\theta = \frac{y}{x} = \frac{-15}{8}$

$\sin\theta < 0$ and $\tan\theta < 0 \implies \theta$ is in Quadrant IV $\implies$
$y < 0$ and $x > 0$.

$x = 8, y = -15, r = 17$

$\sin\theta = \frac{y}{r} = -\frac{15}{17}$ $\qquad$ $\csc\theta = \frac{r}{y} = -\frac{17}{15}$

$\cos\theta = \frac{x}{r} = \frac{8}{17}$ $\qquad$ $\sec\theta = \frac{r}{x} = \frac{17}{8}$

$\tan\theta = \frac{y}{x} = -\frac{15}{8}$ $\qquad$ $\cot\theta = \frac{x}{y} = -\frac{8}{15}$

18. $\cos\theta = \frac{x}{r} = \frac{8}{17} \implies y = |15|$

$\tan\theta < 0 \implies y = -15$

$\sin\theta = \frac{y}{r} = \frac{-15}{17} = -\frac{15}{17}$ $\qquad$ $\csc\theta = -\frac{17}{15}$

$\cos\theta = \frac{x}{r} = \frac{8}{17}$ $\qquad$ $\sec\theta = \frac{17}{8}$

$\tan\theta = \frac{y}{x} = \frac{-15}{8} = -\frac{15}{8}$ $\qquad$ $\cot\theta = -\frac{8}{15}$

19. $\cot\theta = \frac{x}{y} = -\frac{3}{1} = \frac{3}{-1}$

$\cos\theta > 0 \implies \theta$ is in Quadrant IV $\implies x$ is positive;
$x = 3, y = -1, r = \sqrt{10}$

$\sin\theta = \frac{y}{r} = -\frac{\sqrt{10}}{10}$ $\qquad \csc\theta = \frac{r}{y} = -\sqrt{10}$

$\cos\theta = \frac{x}{r} = \frac{3\sqrt{10}}{10}$ $\qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{10}}{3}$

$\tan\theta = \frac{y}{x} = -\frac{1}{3}$ $\qquad \cot\theta = \frac{x}{y} = -3$

20. $\csc\theta = \frac{r}{y} = \frac{4}{1} \implies x = \left|\sqrt{15}\right|$

$\cot\theta < 0 \implies x = -\sqrt{15}$

$\sin\theta = \frac{y}{r} = \frac{1}{4}$ $\qquad \csc\theta = 4$

$\cos\theta = \frac{x}{r} = -\frac{\sqrt{15}}{4}$ $\qquad \sec\theta = -\frac{4\sqrt{15}}{15}$

$\tan\theta = \frac{y}{x} = -\frac{\sqrt{15}}{15}$ $\qquad \cot\theta = -\sqrt{15}$

21. $\sec\theta = \frac{r}{x} = \frac{2}{-1} \implies y^2 = 4 - 1 = 3$

$\sin\theta > 0 \implies \theta$ is in Quadrant II $\implies y = \sqrt{3}$

$\sin\theta = \frac{y}{r} = \frac{\sqrt{3}}{2}$ $\qquad \csc\theta = \frac{r}{y} = \frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{x}{r} = -\frac{1}{2}$ $\qquad \sec\theta = \frac{r}{x} = -2$

$\tan\theta = \frac{y}{x} = -\sqrt{3}$ $\qquad \cot\theta = \frac{x}{y} = -\frac{\sqrt{3}}{3}$

22. $\sin\theta = 0 \implies \theta = 0 + \pi n$

$\sec\theta = -1 \implies \theta = \pi + 2\pi n$

$y = 0, x = -r$

$\sin\theta = 0$ $\qquad \csc\theta = \frac{r}{y}$ is undefined

$\cos\theta = \frac{x}{r} = \frac{-r}{r} = -1$ $\qquad \sec\theta = \frac{r}{x} = -1$

$\tan\theta = \frac{y}{x} = 0$ $\qquad \cot\theta = \frac{x}{y}$ is undefined

23. $\cot\theta$ is undefined, $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2} \implies y = 0 \implies \theta = \pi$

$\sin\pi = 0$ $\qquad \csc\pi$ is undefined

$\cos\pi = -1$ $\qquad \sec\pi = -1$

$\tan\pi = 0$ $\qquad \cot\pi$ is undefined

24. $\tan\theta$ is undefined $\implies \theta = n\pi + \frac{\pi}{2}$

$\pi \le \theta \le 2\pi \implies \theta = \frac{3\pi}{2}, x = 0, y = -r$

$\sin\theta = \frac{y}{r} = \frac{-r}{r} = -1$ $\qquad \csc\theta = \frac{r}{y} = -1$

$\cos\theta = \frac{x}{r} = \frac{0}{r} = 0$ $\qquad \sec\theta = \frac{r}{x}$ is undefined.

$\tan\theta = \frac{y}{x}$ is undefined. $\qquad \cot\theta = \frac{x}{y} = \frac{0}{y} = 0$

25. To find a point on the terminal side of θ use any point on the line $y = -x$ that lies in Quadrant II. $(-1, 1)$ is one such point.

$x = -1, y = 1, r = \sqrt{2}$

$\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos\theta = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\tan\theta = -1$

$\csc\theta = \sqrt{2}$

$\sec\theta = -\sqrt{2}$

$\cot\theta = -1$

26. Let $x > 0$.

$\left(-x, -\frac{1}{3}x\right)$, Quadrant III

$r = \sqrt{x^2 + \frac{1}{9}x^2} = \frac{\sqrt{10}x}{3}$

$\sin\theta = \frac{y}{r} = \frac{(-1/3)x}{(\sqrt{10}x)/3} = -\frac{\sqrt{10}}{10}$

$\cos\theta = \frac{x}{r} = \frac{-x}{(\sqrt{10}x)/3} = -\frac{3\sqrt{10}}{10}$

$\tan\theta = \frac{y}{x} = \frac{(-1/3)x}{-x} = \frac{1}{3}$

$\csc\theta = \frac{r}{y} = \frac{(\sqrt{10}x)/3}{(-1/3)x} = -\sqrt{10}$

$\sec\theta = \frac{r}{x} = \frac{(\sqrt{10}x)/3}{-x} = -\frac{\sqrt{10}}{3}$

$\cot\theta = \frac{x}{y} = \frac{-x}{(-1/3)x} = 3$

27. To find a point on the terminal side of θ, use any point on the line $y = 2x$ that lies in Quadrant III. $(-1, -2)$ is one such point.

$x = -1, y = -2, r = \sqrt{5}$

$\sin\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$

$\cos\theta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

$\tan\theta = \frac{-2}{-1} = 2$

$\csc\theta = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$

$\sec\theta = \frac{\sqrt{5}}{-1} = -\sqrt{5}$

$\cot\theta = \frac{-1}{-2} = \frac{1}{2}$

28. Let $x > 0$.

$4x + 3y = 0 \Longrightarrow y = -\frac{4}{3}x$

$\left(x, -\frac{4}{3}x\right)$, Quadrant IV

$r = \sqrt{x^2 + \frac{16}{9}x^2} = \frac{5}{3}x$

$\sin\theta = \frac{y}{r} = \frac{(-4/3)x}{(5/3)x} = -\frac{4}{5}$ $\qquad \csc\theta = -\frac{5}{4}$

$\cos\theta = \frac{x}{r} = \frac{x}{(5/3)x} = \frac{3}{5}$ $\qquad \sec\theta = \frac{5}{3}$

$\tan\theta = \frac{y}{x} = \frac{(-4/3)x}{x} = -\frac{4}{3}$ $\qquad \tan\theta = -\frac{3}{4}$

29. $(x, y) = (-1, 0), r = 1$

$\sin\pi = \frac{y}{r} = 0$

30. $\csc\frac{3\pi}{2} = \frac{r}{y} = \frac{1}{-1} = -1$

since $\frac{3\pi}{2}$ corresponds to $(0, -1)$.

31. $(x, y) = (0, -1), r = 1$

$\sec\frac{3\pi}{2} = \frac{r}{x} = \frac{1}{0} \Longrightarrow$ undefined

32. $\sec\pi = \frac{r}{x} = \frac{1}{-1} = -1$

since $\frac{3\pi}{2}$ corresponds to $(-1, 0)$.

33. $(x, y) = (0, 1), r = 1$

$\sin\frac{\pi}{2} = \frac{y}{r} = 1$

34. $\cot\pi = \frac{x}{y} = \frac{-1}{0}$ (undefined)

since π corresponds to $(-1, 0)$.

35. $(x, y) = (-1, 0), r = 1$

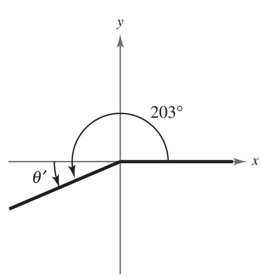
$\csc\pi = \frac{r}{y} = \frac{1}{0} \Longrightarrow$ undefined

36. $\cot\frac{\pi}{2} = \frac{x}{y} = \frac{0}{1} = 0$

since $\frac{\pi}{2}$ corresponds to $(0, 1)$.

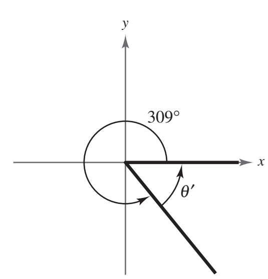
37. $\theta = 203°$

$\theta' = 203° - 180° = 23°$



38. $\theta = 309°$

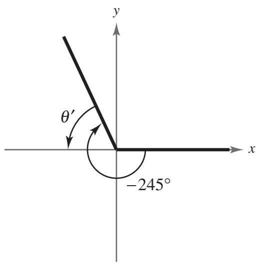
$\theta' = 360° - 309° = 51°$



39. $\theta = -245°$

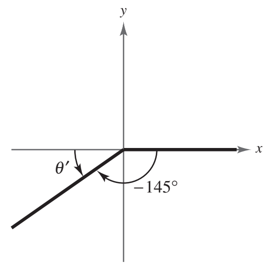
$360° - 245° = 115°$ (coterminal angle)

$\theta' = 180° - 115° = 65°$



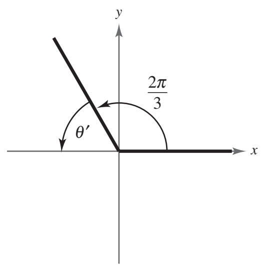
40. $\theta = -145°$ is coterminal with $215°$.

$\theta' = 215° - 180° = 35°$



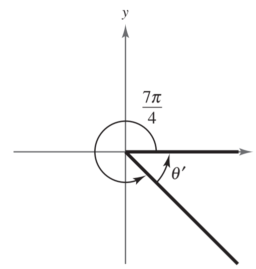
41. $\theta = \dfrac{2\pi}{3}$

$\theta' = \pi - \dfrac{2\pi}{3} = \dfrac{\pi}{3}$



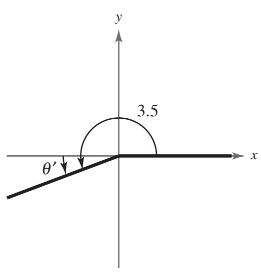
42. $\theta = \dfrac{7\pi}{4}$

$\theta' = 2\pi - \dfrac{7\pi}{4} = \dfrac{\pi}{4}$



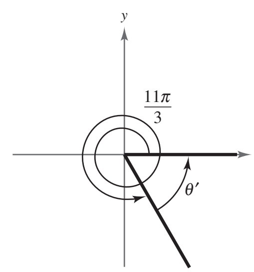
43. $\theta = 3.5$

$\theta' = 3.5 - \pi$



44. $\theta = \dfrac{11\pi}{3}$ is coterminal with $\dfrac{5\pi}{3}$.

$\theta' = 2\pi - \dfrac{5\pi}{3} = \dfrac{\pi}{3}$



45. $\theta = 225°$, $\theta' = 360° - 225° = 45°$, Quadrant III

$\sin 225° = -\sin 45° = -\dfrac{\sqrt{2}}{2}$

$\cos 225° = -\cos 45° = -\dfrac{\sqrt{2}}{2}$

$\tan 225° = \tan 45° = 1$

46. $\theta = 300°$, $\theta' = 360° - 300° = 60°$, Quadrant IV

$\sin 300° = -\sin 60° = -\dfrac{\sqrt{3}}{2}$

$\cos 300° = \cos 60° = \dfrac{1}{2}$

$\tan 300° = -\tan 60° = -\sqrt{3}$

47. $\theta = 750°$ is coterminal with $30°$.

$\theta' = 30°$, Quadrant I

$\sin 750° = \sin 30° = \dfrac{1}{2}$

$\cos 750° = \cos 30° = \dfrac{\sqrt{3}}{2}$

$\tan 750° = \tan 30° = \dfrac{\sqrt{3}}{3}$

48. $\theta = -405°$ is coterminal with $315°$.

$\theta' = 360° - 315° = 45°$, Quadrant IV

$\sin(-405°) = -\sin 45° = -\dfrac{\sqrt{2}}{2}$

$\cos(-405°) = \cos 45° = \dfrac{\sqrt{2}}{2}$

$\tan(-405°) = -\tan 45° = -1$

49. $\theta = -150°$ is coterminal with $210°$.

$\theta' = 210° - 180° = 30°$, Quadrant III

$\sin(-150°) = -\sin 30° = -\dfrac{1}{2}$

$\cos(-150°) = -\cos 30° = -\dfrac{\sqrt{3}}{2}$

$\tan(-150°) = \tan 30° = \dfrac{\sqrt{3}}{3}$

50. $\theta = -840°$ is coterminal with $240°$.

$\theta' = 240° - 180° = 60°$, Quadrant III

$\sin(-840°) = -\sin 60° = -\dfrac{\sqrt{3}}{2}$

$\cos(-840°) = -\cos 60° = -\dfrac{1}{2}$

$\tan(-840°) = \tan 60° = \sqrt{3}$

51. $\theta = \dfrac{4\pi}{3}$, $\theta' = \dfrac{\pi}{3}$, Quadrant III

$\sin \dfrac{4\pi}{3} = -\sin \dfrac{\pi}{3} = -\dfrac{\sqrt{3}}{2}$

$\cos \dfrac{4\pi}{3} = -\cos \dfrac{\pi}{3} = -\dfrac{1}{2}$

$\tan \dfrac{4\pi}{3} = \tan \dfrac{\pi}{3} = \sqrt{3}$

52. $\theta = \dfrac{\pi}{4}$, $\theta' = \dfrac{\pi}{4}$, Quadrant I

$\sin \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

$\cos \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

$\tan \dfrac{\pi}{4} = 1$

53. $\theta = -\dfrac{\pi}{6}$, $\theta' = \dfrac{\pi}{6}$, Quadrant IV

$\sin\left(-\dfrac{\pi}{6}\right) = -\sin \dfrac{\pi}{6} = -\dfrac{1}{2}$

$\cos\left(-\dfrac{\pi}{6}\right) = \cos \dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

$\tan\left(-\dfrac{\pi}{6}\right) = -\tan \dfrac{\pi}{6} = -\dfrac{\sqrt{3}}{3}$

54. $\theta = -\dfrac{\pi}{2}$ is coterminal with $\dfrac{3\pi}{2}$.

$\sin\left(-\dfrac{\pi}{2}\right) = \sin \dfrac{3\pi}{2} = -1$

$\cos\left(-\dfrac{\pi}{2}\right) = \cos \dfrac{3\pi}{2} = 0$

$\tan\left(-\dfrac{\pi}{2}\right) = \tan \dfrac{3\pi}{2}$ is undefined.

55. $\theta = \dfrac{11\pi}{4}$ is coterminal with $\dfrac{3}{4}\pi$.

$\theta' = \pi - \dfrac{3}{4}\pi = \dfrac{\pi}{4}$, Quadrant II

$\sin \dfrac{11\pi}{4} = \sin \dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

$\cos \dfrac{11\pi}{4} = -\cos \dfrac{\pi}{4} = -\dfrac{\sqrt{2}}{2}$

$\tan \dfrac{11\pi}{4} = -\tan \dfrac{\pi}{4} = -1$

56. $\theta = \dfrac{10\pi}{3}$ is coterminal with $\dfrac{4\pi}{3}$.

$\theta' = \dfrac{4\pi}{3} - \pi = \dfrac{\pi}{3}$, Quadrant III

$\sin \dfrac{10\pi}{3} = -\sin \dfrac{\pi}{3} = -\dfrac{\sqrt{3}}{2}$

$\cos \dfrac{10\pi}{3} = -\cos \dfrac{\pi}{3} = -\dfrac{1}{2}$

$\tan \dfrac{10\pi}{3} = \tan \dfrac{\pi}{3} = \sqrt{3}$

57. $\theta = -\dfrac{3\pi}{2}$ is coterminal with $\dfrac{\pi}{2}$, $\theta' = \dfrac{\pi}{2}$.

$\sin\left(-\dfrac{3\pi}{2}\right) = \sin \dfrac{\pi}{2} = 1$

$\cos\left(-\dfrac{3\pi}{2}\right) = \cos \dfrac{\pi}{2} = 0$

$\tan\left(-\dfrac{3\pi}{2}\right) = \tan \dfrac{\pi}{2}$ which is undefined.

58. $\theta = -\dfrac{25\pi}{4}$ is coterminal with $\dfrac{7\pi}{4}$.

$\theta' = 2\pi - \dfrac{7\pi}{4} = \dfrac{\pi}{4}$ in Quadrant IV.

$\sin\left(-\dfrac{25\pi}{4}\right) = -\sin\left(\dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

$\cos\left(-\dfrac{25\pi}{4}\right) = \cos\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

$\tan\left(-\dfrac{25\pi}{4}\right) = -\tan\left(\dfrac{\pi}{4}\right) = -1$

59. $\sin\theta = -\dfrac{3}{5}$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = 1 - \left(-\dfrac{3}{5}\right)^2$

$\cos^2\theta = 1 - \dfrac{9}{25}$

$\cos^2\theta = \dfrac{16}{25}$

$\cos\theta > 0$ in Quadrant IV.

$\cos\theta = \dfrac{4}{5}$

60. $\cot\theta = -3$

$1 + \cot^2\theta = \csc^2\theta$

$1 + (-3)^2 = \csc^2\theta$

$10 = \csc^2\theta$

$\csc\theta > 0$ in Quadrant II.

$\sqrt{10} = \csc\theta$

$\csc\theta = \dfrac{1}{\sin\theta}$

$\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{\sqrt{10}} = \dfrac{\sqrt{10}}{10}$

61. $\tan\theta = \dfrac{3}{2}$

$\sec^2\theta = 1 + \tan^2\theta$

$\sec^2\theta = 1 + \left(\dfrac{3}{2}\right)^2$

$\sec^2\theta = 1 + \dfrac{9}{4}$

$\sec^2\theta = \dfrac{13}{4}$

$\sec\theta < 0$ in Quadrant III.

$\sec\theta = -\dfrac{\sqrt{13}}{2}$

62. $\csc\theta = -2$

$1 + \cot^2\theta = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

$\cot^2\theta = (-2)^2 - 1$

$\cot^2\theta = 3$

$\cot\theta < 0$ in Quadrant IV.

$\cot\theta = -\sqrt{3}$

63. $\cos\theta = \dfrac{5}{8}$

$\cos\theta = \dfrac{1}{\sec\theta} \Rightarrow \sec\theta = \dfrac{1}{\cos\theta}$

$\sec\theta = \dfrac{1}{5/8} = \dfrac{8}{5}$

64. $\sec\theta = -\dfrac{9}{4}$

$1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$\tan^2\theta = \left(-\dfrac{9}{4}\right)^2 - 1$

$\tan^2\theta = \dfrac{65}{16}$

$\tan\theta > 0$ in Quadrant III.

$\tan\theta = \dfrac{\sqrt{65}}{4}$

65. $\sin 10° \approx 0.1736$

66. $\sec 225° = \dfrac{1}{\cos 225°} \approx -1.4142$

67. $\cos(-110°) \approx -0.3420$

68. $\csc(-330°) = \dfrac{1}{\sin(-330°)} = 2.0000$

69. $\tan 304° \approx -1.4826$

70. $\cot 178° = \dfrac{1}{\tan 178°} = -28.6363$

71. $\sec 72° = \dfrac{1}{\cos 72°} \approx 3.2361$

72. $\tan(-188°) = -0.1405$

73. $\tan 4.5 \approx 4.6373$

74. $\cot 1.35 = \dfrac{1}{\tan 1.35} \approx 0.2245$

75. $\tan\dfrac{\pi}{9} \approx 0.3640$

76. $\tan\left(-\dfrac{\pi}{9}\right) \approx -0.3640$

77. $\sin(-0.65) \approx -0.6052$

78. $\sec 0.29 = \dfrac{1}{\cos 0.29} \approx 1.0436$

79. $\cot\left(-\dfrac{11\pi}{8}\right) = \dfrac{1}{\tan\left(-\dfrac{11\pi}{8}\right)} \approx -0.4142$

80. $\csc\left(-\dfrac{15\pi}{14}\right) = \dfrac{1}{\sin\left(-\dfrac{15\pi}{14}\right)} \approx 4.4940$

81. (a) $\sin\theta = \frac{1}{2} \implies$ reference angle is 30° or $\frac{\pi}{6}$ and θ is in Quadrant I or Quadrant II.

Values in degrees: 30°, 150°

Values in radians: $\frac{\pi}{6}, \frac{5\pi}{6}$

(b) $\sin\theta = -\frac{1}{2} \implies$ reference angle is 30° or $\frac{\pi}{6}$ and θ is in Quadrant III or Quadrant IV.

Values in degrees: 210°, 330°

Values in radians: $\frac{7\pi}{6}, \frac{11\pi}{6}$

82. (a) $\cos\theta = \frac{\sqrt{2}}{2} \implies$ reference angle is 45° or $\frac{\pi}{4}$ and θ is in Quadrant I or IV.

Values in degrees: 45°, 315°

Values in radians: $\frac{\pi}{4}, \frac{7\pi}{4}$

(b) $\cos\theta = -\frac{\sqrt{2}}{2} \implies$ reference angle is 45° or $\frac{\pi}{4}$ and θ is in Quadrant II or III.

Values in degrees: 135°, 225°

Values in radians: $\frac{3\pi}{4}, \frac{5\pi}{4}$

83. (a) $\csc\theta = \frac{2\sqrt{3}}{3} \implies$ reference angle is 60° or $\frac{\pi}{3}$ and θ is in Quadrant I or Quadrant II.

Values in degrees: 60°, 120°

Values in radians: $\frac{\pi}{3}, \frac{2\pi}{3}$

(b) $\cot\theta = -1 \implies$ reference angle is 45° or $\frac{\pi}{4}$ and θ is in Quadrant II or Quadrant IV.

Values in degrees: 135°, 315°

Values in radians: $\frac{3\pi}{4}, \frac{7\pi}{4}$

84. (a) $\sec\theta = 2 \implies$ reference angle is 60° or $\frac{\pi}{3}$ and θ is in Quadrant I or IV.

Values in degrees: 60°, 300°

Values in radians: $\frac{\pi}{3}, \frac{5\pi}{3}$

(b) $\sec\theta = -2 \implies$ reference angle is 60° or $\frac{\pi}{3}$ and θ is in Quadrant II or III.

Values in degrees: 120°, 240°

Values in radians: $\frac{2\pi}{3}, \frac{4\pi}{3}$

85. (a) $\tan\theta = 1 \implies$ reference angle is 45° or $\frac{\pi}{4}$ and θ is in Quadrant I or Quadrant III.

Values in degrees: 45°, 225°

Values in radians: $\frac{\pi}{4}, \frac{5\pi}{4}$

(b) $\cot\theta = -\sqrt{3} \implies$ reference angle is 30° or $\frac{\pi}{6}$ and θ is in Quadrant II or Quadrant IV.

Values in degrees: 150°, 330°

Values in radians: $\frac{5\pi}{6}, \frac{11\pi}{6}$

86. (a) $\sin\theta = \frac{\sqrt{3}}{2} \implies$ reference angle is $60°$ or $\frac{\pi}{3}$ and θ is in Quadrant I or II.

Values in degrees: $60°, 120°$

Values in radians: $\frac{\pi}{3}, \frac{2\pi}{3}$

(b) $\sin\theta = -\frac{\sqrt{3}}{2} \implies$ reference angle is $60°$ or $\frac{\pi}{3}$ and θ is in Quadrant III or IV.

Values in degrees: $240°, 300°$

Values in radians: $\frac{4\pi}{3}, \frac{5\pi}{3}$

87. (a) New York City:

$N \approx 22.099 \sin(0.522t - 2.219) + 55.008$

Fairbanks:

$F \approx 36.641 \sin(0.502t - 1.831) + 25.610$

(b)

Month	New York City	Fairbanks
February	34.6°	−1.4°
March	41.6°	13.9°
May	63.4°	48.6°
June	72.5°	59.5°
August	75.5°	55.6°
September	68.6°	41.7°
November	46.8°	6.5°

(c) The periods are about the same for both models, approximately 12 months.

88. $S = 23.1 + 0.442t + 4.3\cos\frac{\pi t}{6}$

(a) For February 2006, $t = 2$.

$$S = 23.1 + 0.442(2) + 4.3\cos\frac{\pi(2)}{6} \approx 26{,}134 \text{ units}$$

(b) For February 2007, $t = 14$.

$$S = 23.1 + 0.442(14) + 4.3\cos\frac{\pi(14)}{6} \approx 31{,}438 \text{ units}$$

(c) For June 2006, $t = 6$.

$$S = 23.1 + 0.442(6) + 4.3\cos\frac{\pi(6)}{6} \approx 21{,}452 \text{ units}$$

(d) For June 2007, $t = 18$.

$$S = 23.1 + 0.442(18) + 4.3\cos\frac{\pi(18)}{6} \approx 26{,}756 \text{ units}$$

89. $y(t) = 2\cos 6t$

(a) $y(0) = 2\cos 0 = 2$ centimeters

(b) $y\left(\frac{1}{4}\right) = 2\cos\left(\frac{3}{2}\right) \approx 0.14$ centimeter

(c) $y\left(\frac{1}{2}\right) = 2\cos 3 \approx -1.98$ centimeters

90. $y(t) = 2e^{-t}\cos 6t$

(a) $t = 0$

$y(0) = 2e^{-0}\cos 0 = 2$ centimeters

(b) $t = \frac{1}{4}$

$y\left(\frac{1}{4}\right) = 2e^{-1/4}\cos\left(6 \cdot \frac{1}{4}\right) \approx 0.11$ centimeters

(c) $t = \frac{1}{2}$

$y\left(\frac{1}{2}\right) = 2e^{-1/2}\cos\left(6 \cdot \frac{1}{2}\right) \approx -1.2$ centimeters

91. $I = 5e^{-2t}\sin t$

$I(0.7) = 5e^{-1.4}\sin 0.7 \approx 0.79$ ampere

92. $\sin\theta = \dfrac{6}{d} \implies d = \dfrac{6}{\sin\theta}$

(a) $\theta = 30°$

$$d = \frac{6}{\sin 30°} = \frac{6}{1/2} = 12 \text{ miles}$$

(b) $\theta = 90°$

$$d = \frac{6}{\sin 90°} = \frac{6}{1} = 6 \text{ miles}$$

(c) $\theta = 120°$

$$d = \frac{6}{\sin 120°} \approx 6.9 \text{ miles}$$

93. False. In each of the four quadrants, the sign of the secant function and the cosine function will be the same since they are reciprocals of each other.

94. False. For example, if $n = 1$ and $\theta = 225°$, $0 \le 135 \le 360$, but $360°n - \theta = 135°$ is not the reference angle. The reference angle would be 45°. For θ in Quadrant II, $\theta' = 180° - \theta$. For θ in Quadrant III, $\theta' = \theta - 180°$. For θ in Quadrant IV, $\theta' = 360° - \theta$.

95. As θ increases from 0° to 90°, x decreases from 12 cm to 0 cm and y increases from 0 cm to 12 cm.

Therefore, $\sin\theta = \dfrac{y}{12}$ increases from 0 to 1 and $\cos\theta = \dfrac{x}{12}$ decreases from 1 to 0. Thus,

$\tan\theta = \dfrac{y}{x}$ increases without bound, and when $\theta = 90°$ the tangent is undefined.

96. Determine the trigonometric function of the reference angle and prefix the appropriate sign.

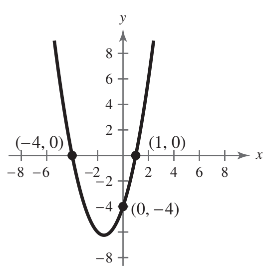
97. $y = x^2 + 3x - 4 = (x + 4)(x - 1)$

x-intercepts: $(-4, 0), (1, 0)$

y-intercept: $(0, -4)$

No asymptotes

Domain: All real numbers x



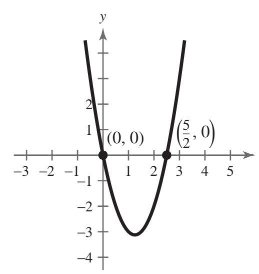
98. $y = 2x^2 - 5x = x(2x - 5)$

x-intercepts: $(0, 0), \left(\frac{5}{2}, 0\right)$

y-intercepts: $(0, 0)$

No asymptotes

Domain: All real numbers x



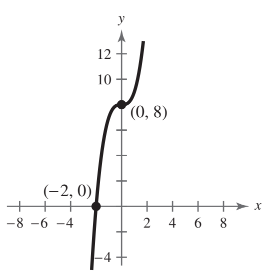
99. $f(x) = x^3 + 8$

x-intercept: $(-2, 0)$

y-intercept: $(0, 8)$

No asymptotes

Domain: All real numbers x



100. $g(x) = x^4 + 2x^2 - 3 = (x^2 + 3)(x^2 - 1)$

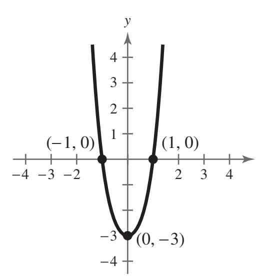
$$= (x^2 + 3)(x + 1)(x - 1)$$

x-intercepts: $(-1, 0), (1, 0)$

y-intercepts: $(0, -3)$

No asymptotes

Domain: All real numbers x



101. $f(x) = \dfrac{x - 7}{x^2 + 4x + 4} = \dfrac{x - 7}{(x + 2)^2}$

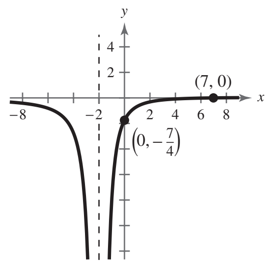
x-intercept: $(7, 0)$

y-intercept: $\left(0, -\dfrac{7}{4}\right)$

Vertical asymptote: $x = -2$

Horizontal asymptote: $y = 0$

Domain: All real numbers except $x = -2$



102. $h(x) = \dfrac{x^2 - 1}{x + 5} = \dfrac{(x + 1)(x - 1)}{x + 5}$

x-intercepts: $(-1, 0)$, $(1, 0)$,

To find the y-intercept, let $x = 0$: $\dfrac{0^2 - 1}{0 + 5} = -\dfrac{1}{5}$

y-intercept: $\left(0, -\dfrac{1}{5}\right)$

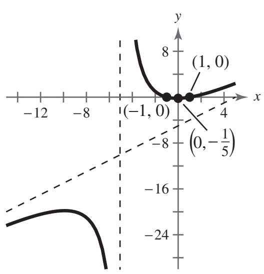
Vertical asymptote: $x = -5$

To find the slant asymptote, use long division:

$$\frac{x^2 - 1}{x + 5} = x - 5 + \frac{24}{x + 5}$$

Slant asymptote: $y = x - 5$

Domain: All real numbers except $x = -5$



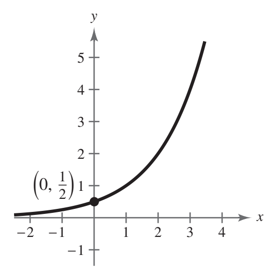
103. $y = 2^{x-1}$

y-intercept: $\left(0, \dfrac{1}{2}\right)$

Horizontal asymptote: $y = 0$

Domain: All real numbers x

x	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



104. $y = 3^{x+1} + 2$

This is an exponential function (always positive) translated two units upward. There are no x-intercepts.

To find the y-intercept, let $x = 0$:

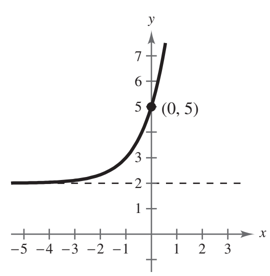
$y = 3^{0+1} + 2 = 3 + 2 = 5$

y-intercepts: $(0, 5)$

The horizontal asymptote is the horizontal asymptote of $y = 3^{x+1}$ translated two units upward.

Horizontal asymptote: $y = 2$

Domain: All real numbers x

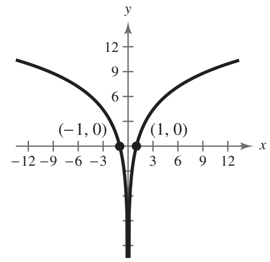


105. $y = \ln x^4$

Domain: All real numbers except $x = 0$

x-intercepts: $(\pm 1, 0)$

Vertical asymptote: $x = 0$



106. $y = \log_{10}(x + 2)$

To find the x-intercept, let $y = 0$:

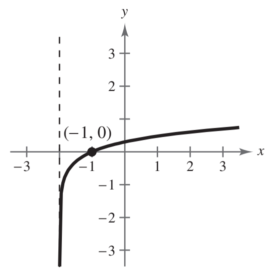
$0 = \log_{10}(x + 2) \implies 10^{\circ} = x + 2 \implies x = -1$

x-intercepts: $(-1, 0)$

To find the y-intercept, let $x = 0$:

$y = \log_{10}(x + 2) = \log_{10} 2 \approx 0.301$

y-intercepts: $(0, 0.301)$



The vertical asymptote is the horizontal asymptote of $y = \log_{10} x$ translated two units to the left.

Vertical asymptote: $x = -2$

Domain: All real numbers x such that $x > -2$

Section 4.5 Graphs of Sine and Cosine Functions

- You should be able to graph $y = a\sin(bx - c)$ and $y = a\cos(bx - c)$. (Assume $b > 0$.)
- Amplitude: $|a|$
- Period: $\dfrac{2\pi}{b}$
- Shift: Solve $bx - c = 0$ and $bx - c = 2\pi$.
- Key increments: $\dfrac{1}{4}$ (period)

Vocabulary Check

1. cycle

2. amplitude

3. $\dfrac{2\pi}{b}$

4. phase shift

5. vertical shift

1. $y = 3 \sin 2x$

Period: $\dfrac{2\pi}{2} = \pi$

Amplitude: $|3| = 3$

2. $y = 2 \cos 3x$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{3}$

Amplitude: $|a| = 2$

3. $y = \dfrac{5}{2} \cos \dfrac{x}{2}$

Period: $\dfrac{2\pi}{1/2} = 4\pi$

Amplitude: $\left|\dfrac{5}{2}\right| = \dfrac{5}{2}$

4. $y = -3 \sin \dfrac{x}{3}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{1/3} = 6\pi$

Amplitude: $|a| = |-3| = 3$

5. $y = \dfrac{1}{2} \sin \dfrac{\pi x}{3}$

Period: $\dfrac{2\pi}{\pi/3} = 6$

Amplitude: $\left|\dfrac{1}{2}\right| = \dfrac{1}{2}$

6. $y = \dfrac{3}{2} \cos \dfrac{\pi x}{2}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{\pi/2} = 4$

Amplitude: $|a| = \dfrac{3}{2}$

7. $y = -2 \sin x$

Period: $\dfrac{2\pi}{1} = 2\pi$

Amplitude: $|-2| = 2$

8. $y = -\cos \dfrac{2x}{3}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{2/3} = 3\pi$

Amplitude: $|a| = |-1| = 1$

9. $y = 3 \sin 10x$

Period: $\dfrac{2\pi}{10} = \dfrac{\pi}{5}$

Amplitude: $|3| = 3$

10. $y = \dfrac{1}{3} \sin 8x$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{8} = \dfrac{\pi}{4}$

Amplitude: $|a| = \dfrac{1}{3}$

11. $y = \dfrac{1}{2} \cos \dfrac{2x}{3}$

Period: $\dfrac{2\pi}{2/3} = 3\pi$

Amplitude: $\left|\dfrac{1}{2}\right| = \dfrac{1}{2}$

12. $y = \dfrac{5}{2} \cos \dfrac{x}{4}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{1/4} = 8\pi$

Amplitude: $|a| = \dfrac{5}{2}$

13. $y = \dfrac{1}{4} \sin 2\pi x$

Period: $\dfrac{2\pi}{2\pi} = 1$

Amplitude: $\left|\dfrac{1}{4}\right| = \dfrac{1}{4}$

14. $y = \dfrac{2}{3} \cos \dfrac{\pi x}{10}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{\pi/10} = 20$

Amplitude: $|a| = \dfrac{2}{3}$

15. $f(x) = \sin x$

$g(x) = \sin(x - \pi)$

The graph of g is a horizontal shift to the right π units of the graph of f (a phase shift).

16. $f(x) = \cos x$, $g(x) = \cos(x + \pi)$
g is a horizontal shift of f π units to the left.

17. $f(x) = \cos 2x$

$g(x) = -\cos 2x$

The graph of g is a reflection in the x-axis of the graph of f.

18. $f(x) = \sin 3x$, $g(x) = \sin(-3x)$

g is a reflection of f about the y-axis.

19. $f(x) = \cos x$

$g(x) = \cos 2x$

The period of f is twice that of g.

20. $f(x) = \sin x$, $g(x) = \sin 3x$
The period of g is one-third the period of f.

21. $f(x) = \sin 2x$

$f(x) = 3 + \sin 2x$

The graph of g is a vertical shift three units upward of the graph of f.

22. $f(x) = \cos 4x$, $g(x) = -2 + \cos 4x$
g is a vertical shift of f two units downward.

23. The graph of g has twice the amplitude as the graph of f. The period is the same.

24. The period of g is one-third the period of f.

25. The graph of g is a horizontal shift π units to the right of the graph of f.

26. Shift the graph of f two units upward to obtain the graph of g.

27. $f(x) = -2 \sin x$

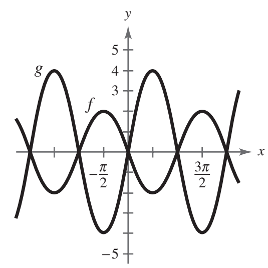
Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{1} = 2\pi$

Amplitude: 2

Symmetry: origin

Key points:	Intercept	Minimum	Intercept	Maximum	Intercept
	$(0, 0)$	$\left(\dfrac{\pi}{2}, -2\right)$	$(\pi, 0)$	$\left(\dfrac{3\pi}{2}, 0\right)$	$(2\pi, 0)$

Since $g(x) = 4 \sin x = (-2)f(x)$, generate key points for the graph of $g(x)$ by multiplying the y-coordinate of each key point of $f(x)$ by -2.



28. $f(x) = \sin x$

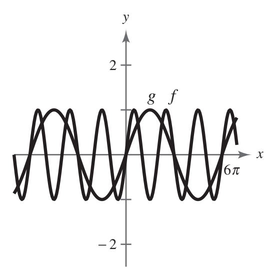
Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{1} = 2\pi$

Amplitude: 1

Symmetry: origin

Key points:	Intercept	Maximum	Intercept	Minimum	Intercept
	$(0, 0)$	$\left(\dfrac{\pi}{2}, 1\right)$	$(\pi, 0)$	$\left(\dfrac{3\pi}{2}, -1\right)$	$(2\pi, 0)$

Since $g(x) = \sin\left(\dfrac{x}{3}\right) = f\left(\dfrac{x}{3}\right)$, the graph of $g(x)$ is the graph of $f(x)$, but stretched horizontally by a factor of 3. Generate key points for the graph of $g(x)$ by multiplying the x-coordinate of each key point of $f(x)$ by 3.



29. $f(x) = \cos x$

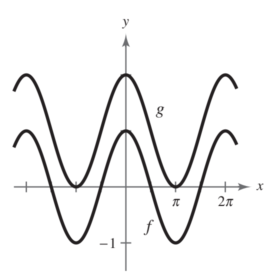
Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{1} = 2\pi$

Amplitude: 1

Symmetry: y-axis

Key points:	Maximum	Intercept	Minimum	Intercept	Maximum
	$(0, 1)$	$\left(\dfrac{\pi}{2}, 0\right)$	$(\pi, -1)$	$\left(\dfrac{3\pi}{2}, 0\right)$	$(2\pi, 1)$

Since $g(x) = 1 + \cos(x) = f(x) + 1$, the graph of $g(x)$ is the graph of $f(x)$, but translated upward by one unit. Generate key points for the graph of $g(x)$ by adding 1 to the y-coordinate of each key point of $f(x)$.

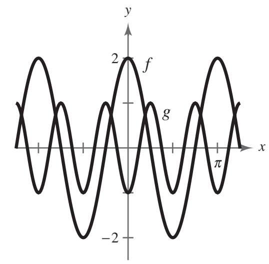


30. $f(x) = 2 \cos 2x$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{2} = \pi$

Amplitude: 2

Symmetry: y-axis



Key points: Maximum	Intercept	Minimum	Intercept	Maximum
$(0, 2)$	$\left(\dfrac{\pi}{4}, 0\right)$	$\left(\dfrac{\pi}{2}, -2\right)$	$\left(\dfrac{3\pi}{4}, 0\right)$	$(\pi, 2)$

Since $g(x) = -\cos 4x = -\frac{1}{2}f(2x)$, the graph of $g(x)$ is the graph of $f(x)$, but

i) shrunk horizontally by a factor of 2,

ii) shrunk vertically by a factor of $\frac{1}{2}$, and

iii) reflected about the x-axis.

Generate key points for the graph of $g(x)$ by

i) dividing the x-coordinate of each key point of $f(x)$ by 2, and

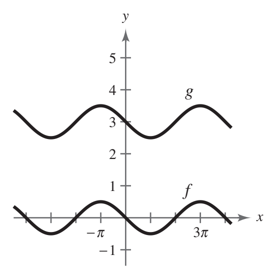
ii) dividing the y-coordinate of each key point of $f(x)$ by -2.

31. $f(x) = -\dfrac{1}{2} \sin \dfrac{x}{2}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{1/2} = 4\pi$

Amplitude: $\dfrac{1}{2}$

Symmetry: origin



Key points: Intercept	Minimum	Intercept	Maximum	Intercept
$(0, 0)$	$\left(\pi, -\dfrac{1}{2}\right)$	$(2\pi, 0)$	$\left(3\pi, \dfrac{1}{2}\right)$	$(4\pi, 0)$

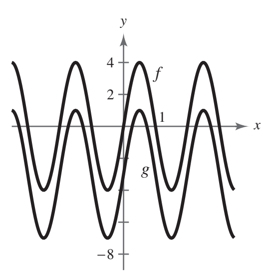
Since $g(x) = 3 - \dfrac{1}{2} \sin \dfrac{x}{2} = 3 - f(x)$, the graph of $g(x)$ is the graph of $f(x)$, but translated upward by three units. Generate key points for the graph of $g(x)$ by adding 3 to the y-coordinate of each key point of $f(x)$.

32. $f(x) = 4 \sin \pi x$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{\pi} = 2$

Amplitude: 4

Symmetry: origin



Key points: Intercept	Maximum	Intercept	Minimum	Intercept
$(0, 0)$	$\left(\dfrac{1}{2}, 2\right)$	$(1, 0)$	$\left(\dfrac{3}{2}, -2\right)$	$(2, 0)$

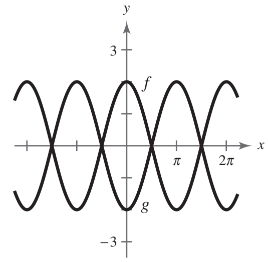
Since $g(x) = 4 \sin \pi x - 3 = f(x) - 3$, the graph of $g(x)$ is the graph of $f(x)$, but translated downward by three units. Generate key points for the graph of $g(x)$ by subtracting 3 from the y-coordinate of each key point of $f(x)$.

33. $f(x) = 2\cos x$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{1} = 2\pi$

Amplitude: 2

Symmetry: y-axis



Key points:	Maximum	Intercept	Minimum	Intercept	Maximum
	$(0, 2)$	$\left(\dfrac{\pi}{2}, 0\right)$	$(\pi, -2)$	$\left(\dfrac{3\pi}{2}, 0\right)$	$(2\pi, 2)$

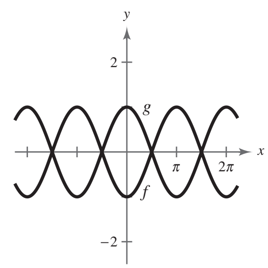
Since $g(x) = 2\cos(x + \pi) = f(x + \pi)$, the graph of $g(x)$ is the graph of $f(x)$, but with a phase shift (horizontal translation) of $-\pi$. Generate key points for the graph of $g(x)$ by shifting each key point of $f(x)$ π units to the left.

34. $f(x) = -\cos x$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{1} = 2\pi$

Amplitude: 1

Symmetry: y-axis



Key points:	Minimum	Intercept	Maximum	Intercept	Minimum
	$(0, -1)$	$\left(\dfrac{\pi}{2}, 0\right)$	$(\pi, 1)$	$\left(\dfrac{3\pi}{2}, 0\right)$	$(2\pi, -1)$

Since $g(x) = -\cos(x - \pi) = f(x - \pi)$, the graph of $g(x)$ is the graph of $f(x)$, but with a phase shift (horizontal translation) of π. Generate key points for the graph of $g(x)$ by shifting each key point of $f(x)$ π units to the right.

35. $y = 3\sin x$

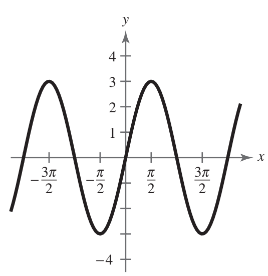
Period: 2π

Amplitude: 3

Key points:

$(0, 0), \left(\dfrac{\pi}{2}, 3\right), (\pi, 0),$

$\left(\dfrac{3\pi}{2}, -3\right), (2\pi, 0)$



36. $y = \dfrac{1}{4}\sin x$

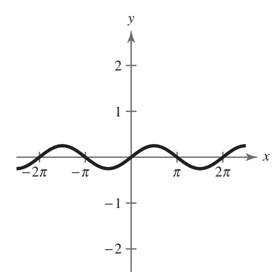
Period: 2π

Amplitude: $\dfrac{1}{4}$

Key points:

$(0, 0), \left(\dfrac{\pi}{2}, \dfrac{1}{4}\right), (\pi, 0),$

$\left(\dfrac{3\pi}{2}, -\dfrac{1}{4}\right), (2\pi, 0)$



37. $y = \dfrac{1}{3}\cos x$

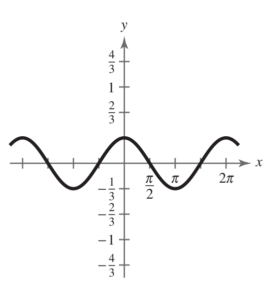
Period: 2π

Amplitude: $\dfrac{1}{3}$

Key points:

$\left(0, \dfrac{1}{3}\right), \left(\dfrac{\pi}{2}, 0\right), \left(\pi, -\dfrac{1}{3}\right),$

$\left(\dfrac{3\pi}{2}, 0\right), \left(2\pi, \dfrac{1}{3}\right)$



38. $y = 4\cos x$

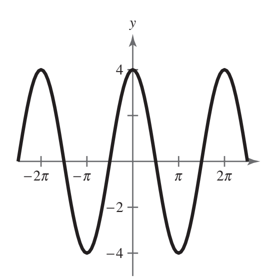
Period: 2π

Amplitude: 4

Key points:

$(0, 4), \left(\dfrac{\pi}{2}, 0\right), (\pi, -4),$

$\left(\dfrac{3\pi}{2}, 0\right), (2\pi, 4)$



39. $y = \cos \dfrac{x}{2}$

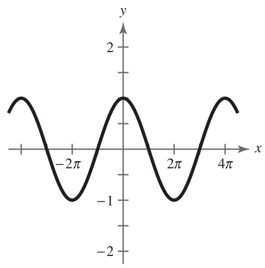
Period: 4π

Amplitude: 1

Key points:

$(0, 1), (\pi, 0), (2\pi, -1),$

$(3\pi, 0), (4\pi, 1)$



40. $y = \sin 4x$

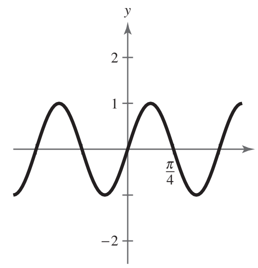
Period: $\dfrac{\pi}{2}$

Amplitude: 1

Key points:

$(0, 0), \left(\dfrac{\pi}{8}, 1\right), \left(\dfrac{\pi}{4}, 0\right),$

$\left(\dfrac{3\pi}{8}, -1\right), \left(\dfrac{\pi}{2}, 0\right)$



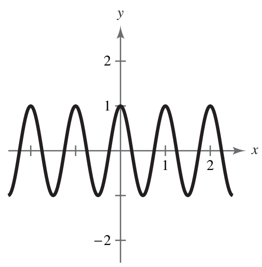
41. $y = \cos 2\pi x$

Period: $\dfrac{2\pi}{2\pi} = 1$

Amplitude: 1

Key points:

$(0, 1), \left(\dfrac{1}{4}, 0\right), \left(\dfrac{1}{2}, -1\right), \left(\dfrac{3}{4}, 0\right)$



42. $y = \sin \dfrac{\pi x}{4}$

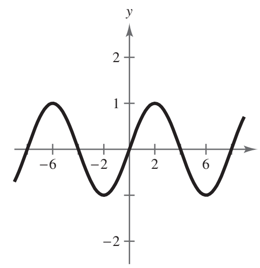
Period: $\dfrac{2\pi}{\pi/4} = 8$

Amplitude: 1

Key points:

$(0, 0), (2, 1), (4, 0),$

$(6, -1), (8, 0)$

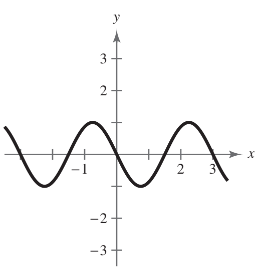


43. $y = -\sin \dfrac{2\pi x}{3}$; $a = -1, b = \dfrac{2\pi}{3}, c = 0$

Period: $\dfrac{2\pi}{2\pi/3} = 3$

Amplitude: 1

Key points: $(0, 0), \left(\dfrac{3}{4}, -1\right), \left(\dfrac{3}{2}, 0\right), \left(\dfrac{9}{4}, 1\right), (3, 0)$



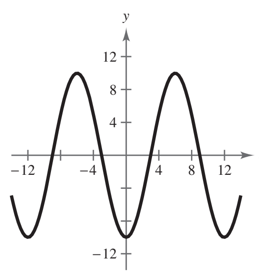
44. $y = -10 \cos \dfrac{\pi x}{6}$

Period: $\dfrac{2\pi}{\pi/6} = 12$

Amplitude: 10

Key points:

$(0, -10), (3, 0), (6, 10), (9, 0), (12, -10)$



45. $y = \sin\left(x - \dfrac{\pi}{4}\right)$; $a = 1, b = 1, c = \dfrac{\pi}{4}$

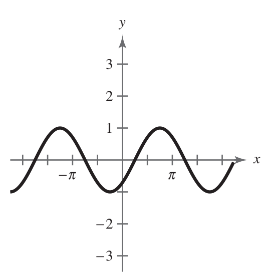
Period: 2π

Amplitude: 1

Shift: Set $x - \dfrac{\pi}{4} = 0$ and $x - \dfrac{\pi}{4} = 2\pi$

$x = \dfrac{\pi}{4}$ $\qquad$ $x = \dfrac{9\pi}{4}$

Key points: $\left(\dfrac{\pi}{4}, 0\right), \left(\dfrac{3\pi}{4}, 1\right), \left(\dfrac{5\pi}{4}, 0\right), \left(\dfrac{7\pi}{4}, -1\right), \left(\dfrac{9\pi}{4}, 0\right)$



46. $y = \sin(x - \pi)$

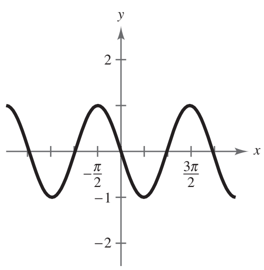
Period: 2π

Amplitude: 1

Shift: Set $x - \pi = 0$ and $x - \pi = 2\pi$

$x = \pi \qquad x = 3\pi$

Key points: $(\pi, 0), \left(\frac{3\pi}{2}, 1\right), (2\pi, 0), \left(\frac{5\pi}{2}, -1\right), (3\pi, 0)$



47. $y = 3\cos(x + \pi)$

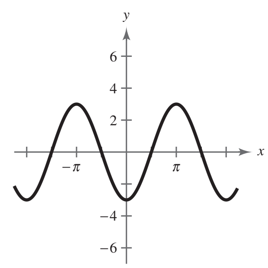
Period: 2π

Amplitude: 3

Shift: Set $x + \pi = 0$ and $x + \pi = 2\pi$

$x = -\pi \qquad x = \pi$

Key points: $(-\pi, 3), \left(-\frac{\pi}{2}, 0\right), (0, -3), \left(\frac{\pi}{2}, 0\right), (\pi, 3)$



48. $y = 4\cos\left(x + \frac{\pi}{4}\right)$

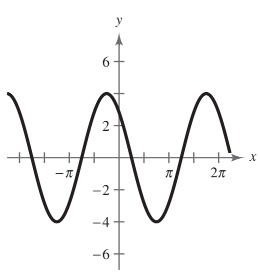
Period: 2π

Amplitude: 4

Shift: Set $x + \frac{\pi}{4} = 0$ and $x + \frac{\pi}{4} = 2\pi$

$x = -\frac{\pi}{4} \qquad x = \frac{7\pi}{4}$

Key points: $\left(-\frac{\pi}{4}, 4\right), \left(\frac{\pi}{4}, 0\right), \left(\frac{3\pi}{4}, -4\right), \left(\frac{5\pi}{4}, 0\right), \left(\frac{7\pi}{4}, 4\right)$

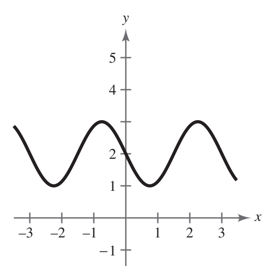


49. $y = 2 - \sin\frac{2\pi x}{3}$

Period: 3

Amplitude: 1

Key points: $(0, 2), \left(\frac{3}{4}, 1\right), \left(\frac{3}{2}, 2\right), \left(\frac{9}{4}, 3\right), (3, 2)$

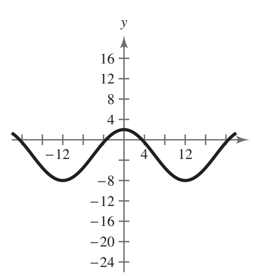


50. $y = -3 + 5\cos\frac{\pi t}{12}$

Period: $\frac{2\pi}{\pi/12} = 24$

Amplitude: 5

Key points: $(0, 2), (6, -3), (12, -8), (18, -3), (24, 2)$



51. $y = 2 + \frac{1}{10} \cos 60\pi x$

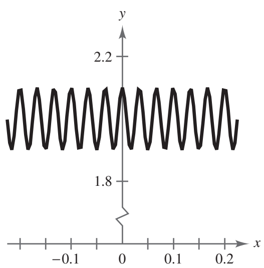
Period: $\frac{2\pi}{60\pi} = \frac{1}{30}$

Amplitude: $\frac{1}{10}$

Vertical shift two units upward

Key points:

$(0, 2.1), \left(\frac{1}{120}, 2\right), \left(\frac{1}{60}, 1.9\right), \left(\frac{1}{40}, 2\right), \left(\frac{1}{30}, 2.1\right)$



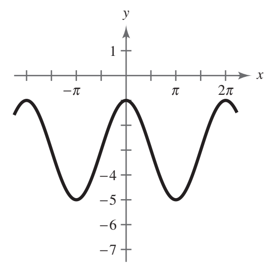
52. $y = 2 \cos x - 3$

Period: 2π

Amplitude: 2

Key points:

$(0, -1), \left(\frac{\pi}{2}, -3\right), (\pi, -5), \left(\frac{3\pi}{2}, -3\right), (2\pi, -1)$



53. $y = 3 \cos(x + \pi) - 3$

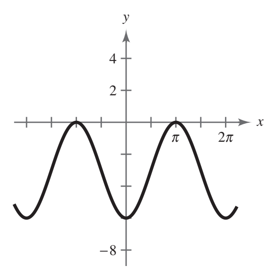
Period: 2π

Amplitude: 3

Shift: Set $x + \pi = 0$ and $x + \pi = 2\pi$

$x = -\pi$ $\qquad$ $x = \pi$

Key points: $(-\pi, 0), \left(-\frac{\pi}{2}, -3\right), (0, -6), \left(\frac{\pi}{2}, -3\right), (\pi, 0)$



54. $y = 4 \cos\left(x + \frac{\pi}{4}\right) + 4$

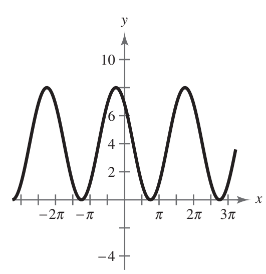
Period: 2π

Amplitude: 4

Shift: Set $x + \frac{\pi}{4} = 0$ and $x + \frac{\pi}{4} = 2\pi$

$x = -\frac{\pi}{4}$ $\qquad$ $x = \frac{7\pi}{4}$

Key points: $\left(-\frac{\pi}{4}, 8\right), \left(\frac{\pi}{4}, 4\right), \left(\frac{3\pi}{4}, 0\right), \left(\frac{5\pi}{4}, 4\right), \left(\frac{7\pi}{4}, 8\right)$



55. $y = \frac{2}{3} \cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$; $a = \frac{2}{3}$, $b = \frac{1}{2}$, $c = \frac{\pi}{4}$

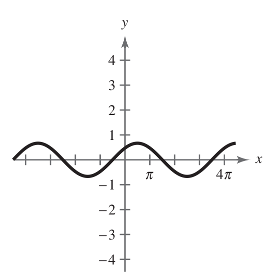
Period: 4π

Amplitude: $\frac{2}{3}$

Shift: $\frac{x}{2} - \frac{\pi}{4} = 0$ and $\frac{x}{2} - \frac{\pi}{4} = 2\pi$

$x = \frac{\pi}{2}$ $\qquad$ $x = \frac{9\pi}{2}$

Key points: $\left(\frac{\pi}{2}, \frac{2}{3}\right), \left(\frac{3\pi}{2}, 0\right), \left(\frac{5\pi}{2}, \frac{-2}{3}\right), \left(\frac{7\pi}{2}, 0\right), \left(\frac{9\pi}{2}, \frac{2}{3}\right)$



56. $y = -3\cos(6x + \pi)$

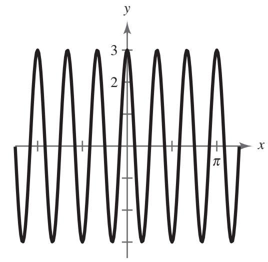
Period: $\dfrac{2\pi}{6} = \dfrac{\pi}{3}$

Amplitude: 3

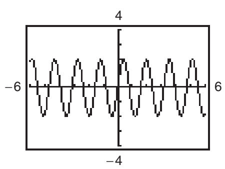
Shift: Set $6x + \pi = 0$ and $6x + \pi = 2\pi$

$x = -\dfrac{\pi}{6}$ $\qquad x = \dfrac{\pi}{6}$

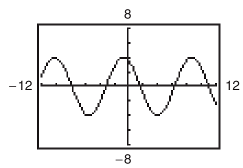
Key points: $\left(-\dfrac{\pi}{6}, -3\right), \left(-\dfrac{\pi}{12}, 0\right), (0, 3), \left(\dfrac{\pi}{12}, 0\right), \left(\dfrac{\pi}{6}, -3\right)$



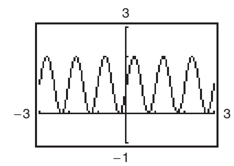
57. $y = -2\sin(4x + \pi)$



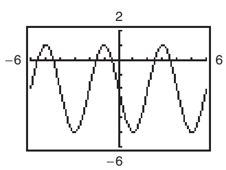
58. $y = -4\sin\left(\dfrac{2}{3}x - \dfrac{\pi}{3}\right)$



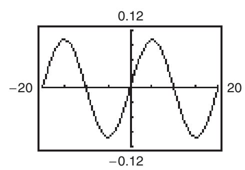
59. $y = \cos\left(2\pi x - \dfrac{\pi}{2}\right) + 1$



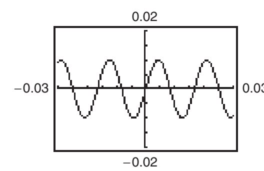
60. $y = 3\cos\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right) - 2$



61. $y = -0.1\sin\left(\dfrac{\pi x}{10} + \pi\right)$



62. $y = \dfrac{1}{100}\sin 120\,\pi t$



63. $f(x) = a\cos x + d$

Amplitude: $\frac{1}{2}[3 - (-1)] = 2 \implies a = 2$

Vertical shift one unit upward of

$g(x) = 2\cos x \implies d = 1$

Thus, $f(x) = 2\cos x + 1$.

64. $f(x) = a\cos x + d$

Amplitude: $\dfrac{1 - (-3)}{2} = 2$

$1 = 2\cos 0 + d$

$d = 1 - 2 = -1$

$a = 2, d = -1$

65. $f(x) = a\cos x + d$

Amplitude: $\frac{1}{2}[8 - 0] = 4$

Since $f(x)$ is the graph of $g(x) = 4\cos x$ reflected in the x-axis and shifted vertically four units upward, we have $a = -4$ and $d = 4$. Thus, $f(x) = -4\cos x + 4$.

66. $f(x) = a\cos x + d$

Amplitude: $\dfrac{-2 - (-4)}{2} = 1$

Reflected in the x-axis: $a = -1$

$-4 = -1\cos 0 + d$

$d = -3$

$a = -1, d = -3$

67. $y = a\sin(bx - c)$

Amplitude: $|a| = |3|$

Since the graph is reflected in the x-axis, we have $a = -3$.

Period: $\dfrac{2\pi}{b} = \pi \Rightarrow b = 2$

Phase shift: $c = 0$

Thus, $y = -3\sin 2x$.

68. $y = a\sin(bx - c)$

Amplitude: $2 \Rightarrow a = 2$

Period: 4π

$\dfrac{2\pi}{b} = 4\pi \Rightarrow b = \dfrac{1}{2}$

Phase shift: $c = 0$

$a = 2, b = \dfrac{1}{2}, c = 0$

69. $y = a\sin(bx - c)$

Amplitude: $a = 2$

Period: $2\pi \Rightarrow b = 1$

Phase shift: $bx - c = 0$ when $x = -\dfrac{\pi}{4}$

$$(1)\left(\frac{-\pi}{4}\right) - c = 0 \Rightarrow c = -\frac{\pi}{4}$$

Thus, $y = 2\sin\left(x + \dfrac{\pi}{4}\right)$.

70. $y = a\sin(bx - c)$

Amplitude: $2 \Rightarrow a = 2$

Period: 2

$\dfrac{2\pi}{b} = 4 \Rightarrow b = \dfrac{\pi}{2}$

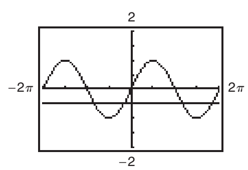
Phase shift: $\dfrac{c}{b} = -1 \Rightarrow c = -\dfrac{\pi}{2}$

$a = 2, b = \dfrac{\pi}{2}, c = -\dfrac{\pi}{2}$

71. $y_1 = \sin x$

$y_2 = -\dfrac{1}{2}$

In the interval $[-2\pi, 2\pi]$,

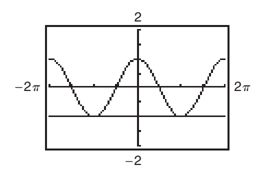


$\sin x = -\dfrac{1}{2}$ when $x = -\dfrac{5\pi}{6}, -\dfrac{\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$.

72. $y_1 = \cos x$

$y_2 = -1$

$y_1 = y_2$ when $x = \pi, -\pi$



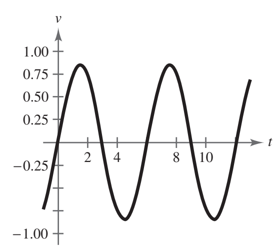
73. $y = 0.85\sin\dfrac{\pi t}{3}$

(a) Time for one cycle $= \dfrac{2\pi}{\pi/3} = 6$ sec

(b) Cycles per min $= \dfrac{60}{6} = 10$ cycles per min

(c) Amplitude: 0.85; Period: 6

Key points: $(0, 0), \left(\dfrac{3}{2}, 0.85\right), (3, 0), \left(\dfrac{9}{2}, -0.85\right), (6, 0)$

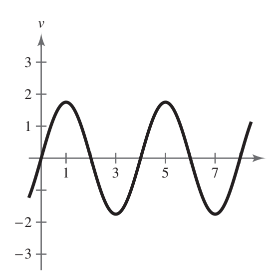


74. $v = 1.75\sin\dfrac{\pi t}{2}$

(a) Period $= \dfrac{2\pi}{\pi/2} = 4$ seconds

(b) $\dfrac{1 \text{ cycle}}{4 \text{ seconds}} \cdot \dfrac{60 \text{ seconds}}{1 \text{ minute}} = 15$ cycles per minute

(c)



75. $y = 0.001 \sin 880\pi t$

(a) Period: $\dfrac{2\pi}{880\pi} = \dfrac{1}{440}$ seconds

(b) $f = \dfrac{1}{p} = 440$ cycles per second

76. $P = 100 - 20 \cos \dfrac{5\pi t}{3}$

(a) Period: $\dfrac{2\pi}{(5\pi)/3} = \dfrac{6}{5}$ seconds

(b) $\dfrac{1 \text{ heartbeat}}{6/5 \text{ seconds}} \cdot \dfrac{60 \text{ seconds}}{1 \text{ minute}} = 50$ heartbeats per minute

77. (a) $a = \frac{1}{2}[\text{high} - \text{low}] = \frac{1}{2}[83.5 - 29.6] = 26.95$

$p = 2[\text{high time} - \text{low time}] = 2[7 - 1] = 12$

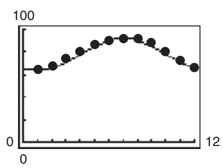
$b = \dfrac{2\pi}{p} = \dfrac{2\pi}{12} = \dfrac{\pi}{6}$

$\dfrac{c}{b} = 7 \implies c = 7\left(\dfrac{\pi}{6}\right) \approx 3.67$

$d = \frac{1}{2}[\text{high} + \text{low}] = \frac{1}{2}[83.5 + 29.6] = 56.55$

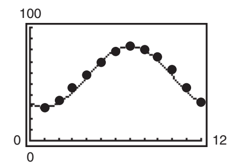
$C(t) = 56.55 + 26.95 \cos\left(\dfrac{\pi t}{6} - 3.67\right)$

(b)



The model is a good fit.

(c)



The model is a good fit.

(d) Tallahassee average maximum: 77.90°

Chicago average maximum: 56.55°

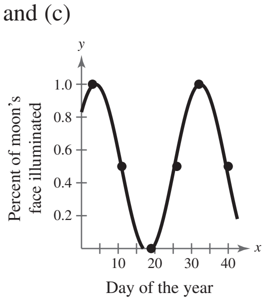
The constant term, d, gives the average maximum temperature.

(e) The period for both models is $\dfrac{2\pi}{\pi/6} = 12$ months.

This is as we expected since one full period is one year.

(f) Chicago has the greater variability in temperature throughout the year. The amplitude, a, determines this variability since it is $\frac{1}{2}$[high temp $-$ low temp].

78. (a) and (c)



Reasonably good fit

(d) Period is 29.6 days.

(e) March 12 $\implies x = 71.$ $y = 0.44 = 44\%$

The Naval observatory says that 50% of the moon's face will be illuminated on March 12, 2007.

(b) Vertical shift: $\dfrac{1}{2} \implies d = \dfrac{1}{2}$

Amplitude: $\dfrac{1}{2} \implies a = \dfrac{1}{2}$

Period: $\dfrac{8 + 8 + 7 + 6 + 8}{5} = 7.4$ (average length of interval in data)

$$\frac{2\pi}{b} = 4(7.4) = 29.6$$

$$b = \frac{2\pi}{29.6} \approx 0.21$$

Horizontal shift: $0.21(3 - 7.4) + C = 0$

$$C = 0.92$$

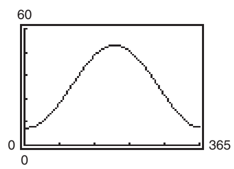
$$y = \frac{1}{2} + \frac{1}{2}\sin(0.21x + 0.92)$$

79. $C = 30.3 + 21.6 \sin\left(\dfrac{2\pi t}{365} + 10.9\right)$

(a) Period $= \dfrac{2\pi}{\frac{2\pi}{365}} = 365$

Yes, this is what is expected because there are 365 days in a year.

(b) The average daily fuel consumption is given by the amount of the vertical shift (from 0) which is given by the constant 30.3.

(c) 

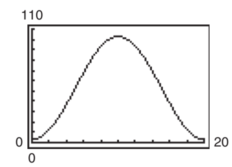
The consumption exceeds 40 gallons per day when $124 < x < 252$.

80. (a) Period $= \dfrac{2\pi}{\left(\frac{\pi}{6}\right)} = 12$ minutes

The wheel takes 12 minutes to revolve once.

(b) Amplitude: 50 feet

The radius of the wheel is 50 feet.

(c) 

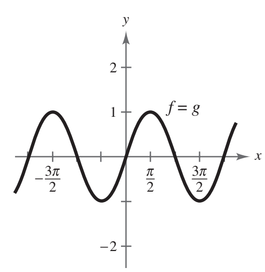
81. False. The graph of $\sin(x + 2\pi)$ is the graph of $\sin(x)$ translated to the *left* by one period, and the graphs are indeed identical.

82. False. $y = \frac{1}{2} \cos 2x$ has an amplitude that is **half** that of $y = \cos x$. For $y = a \cos bx$, the amplitude is $|a|$.

83. True.

Since $\cos x = \sin\left(x + \dfrac{\pi}{2}\right)$, $y = -\cos x = -\sin\left(x + \dfrac{\pi}{2}\right)$, and so is a reflection in the x-axis of $y = \sin\left(x + \dfrac{\pi}{2}\right)$.

84. Answers will vary.

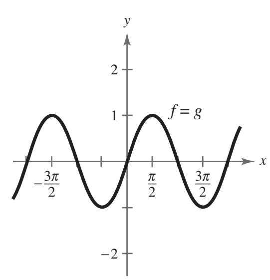
85. 

Since the graphs are the same, the conjecture is that

$\sin(x) = \cos\left(x - \dfrac{\pi}{2}\right)$.

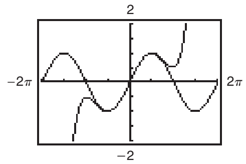
86. $f(x) = \sin x$, $g(x) = -\cos\left(x + \dfrac{\pi}{2}\right)$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	1	0	-1	0
$-\cos\left(x + \frac{\pi}{2}\right)$	0	1	0	-1	0



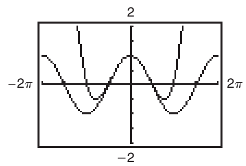
Conjecture: $\sin x = -\cos\left(x + \dfrac{\pi}{2}\right)$

87. (a)



The graphs are nearly the same for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

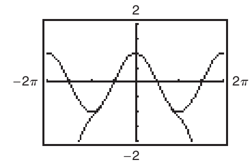
(b)

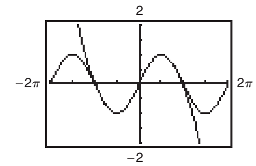


The graphs are nearly the same for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

(c) $\sin x \approx x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \dfrac{x^7}{7!}$

$\cos x \approx 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!}$





The graphs now agree over a wider range, $-\dfrac{3\pi}{4} < x < \dfrac{3\pi}{4}$.

88. (a) $\sin \dfrac{1}{2} \approx \dfrac{1}{2} - \dfrac{(1/2)^3}{3!} + \dfrac{(1/2)^5}{5!} \approx 0.4794$

$\sin \dfrac{1}{2} \approx 0.4794$ (by calculator)

(b) $\sin 1 \approx 1 - \dfrac{1}{3!} + \dfrac{1}{5!} \approx 0.8417$

$\sin 1 \approx 0.8415$ (by calculator)

(c) $\sin \dfrac{\pi}{6} \approx 1 - \dfrac{(\pi/6)^3}{3!} + \dfrac{(\pi/6)^5}{5!} \approx 0.5000$

$\sin \dfrac{\pi}{6} = 0.5$ (by calculator)

(d) $\cos(-0.5) \approx 1 - \dfrac{(-0.5)^2}{2!} + \dfrac{(-0.5)^4}{4!} \approx 0.8776$

$\cos(-0.5) \approx 0.8776$ (by calculator)

(e) $\cos 1 \approx 1 - \dfrac{1}{2!} + \dfrac{1}{4!} \approx 0.5417$

$\cos 1 \approx 0.5403$ (by calculator)

(f) $\cos \dfrac{\pi}{4} \approx 1 - \dfrac{(\pi/4)^2}{2!} + \dfrac{(\pi/4)^2}{4!} = 0.7074$

$\cos \dfrac{\pi}{4} \approx 0.7071$ (by calculator)

The error in the approximation is not the same in each case. The error appears to increase as x moves farther away from 0.

89. $\log_{10} \sqrt{x-2} = \log_{10}(x-2)^{1/2} = \frac{1}{2}\log_{10}(x-2)$

90. $\log_2[x^2(x-3)] = \log_2 x^2 + \log_2(x-3)$

$= 2\log_2 x + \log_2(x-3)$

91. $\ln \dfrac{t^3}{t-1} = \ln t^3 - \ln(t-1) = 3\ln t - \ln(t-1)$

92. $\ln\sqrt{\dfrac{z}{z^2+1}} = \dfrac{1}{2}\ln\left(\dfrac{z}{z^2+1}\right) = \dfrac{1}{2}[\ln z - \ln(z^2+1)]$

$= \dfrac{1}{2}\ln z - \dfrac{1}{2}\ln(z^2+1)$

93. $\frac{1}{2}(\log_{10} x + \log_{10} y) = \frac{1}{2}\log_{10}(xy)$

$= \log_{10}\sqrt{xy}$

94. $2\log_2 x + \log_2(xy) = \log_2 x^2 + \log_2(xy)$

$= \log_2 x^2(xy)$

$= \log_2 x^3 y$

95. $\ln 3x - 4\ln y = \ln 3x - \ln y^4$

$= \ln\left(\dfrac{3x}{y^4}\right)$

96. $\frac{1}{2}(\ln 2x - 2\ln x) + 3\ln x = \frac{1}{2}(\ln 2x - \ln x^2) + \ln x^3$

$$= \frac{1}{2}\left(\ln \frac{2x}{x^2}\right) + \ln x^3$$

$$= \ln \sqrt{\frac{2x}{x^2}} + \ln x^3$$

$$= \ln\left(x^3\sqrt{\frac{2x}{x^2}}\right)$$

$$= \ln\left(x^2\sqrt{2x}\right)$$

97. Answers will vary.

Section 4.6 Graphs of Other Trigonometric Functions

- You should be able to graph
  - $y = a\tan(bx - c)$ $\quad$ $y = a\cot(bx - c)$
  - $y = a\sec(bx - c)$ $\quad$ $y = a\csc(bx - c)$
- When graphing $y = a\sec(bx - c)$ or $y = a\csc(bx - c)$ you should first graph $y = a\cos(bx - c)$ or $y = a\sin(bx - c)$ because
  - (a) The x-intercepts of sine and cosine are the vertical asymptotes of cosecant and secant.
  - (b) The maximums of sine and cosine are the local minimums of cosecant and secant.
  - (c) The minimums of sine and cosine are the local maximums of cosecant and secant.
- You should be able to graph using a damping factor.

Vocabulary Check

1. vertical
2. reciprocal
3. damping
4. π
5. $x \neq n\pi$
6. $(-\infty, -1] \cup [1, \infty)$
7. 2π

1. $y = \sec 2x$

Period: $\frac{2\pi}{2} = \pi$

Matches graph (e).

2. $y = \tan\frac{x}{2}$

Period: $\frac{\pi}{b} = \frac{\pi}{1/2} = 2\pi$

Asymptotes: $x = -\pi, x = \pi$

Matches graph (c).

3. $y = \frac{1}{2}\cot \pi x$

Period: $\frac{\pi}{\pi} = 1$

Matches graph (a).

4. $y = -\csc x$

Period: 2π

Matches graph (d).

5. $y = \frac{1}{2}\sec\frac{\pi x}{2}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{\pi/2} = 4$

Asymptotes: $x = -1, x = 1$

Matches graph (f).

6. $y = -2\sec\frac{\pi x}{2}$

Period: $\frac{2\pi}{b} = \frac{2\pi}{\pi/2} = 4$

Asymptotes: $x = -1, x = 1$

Reflected in x-axis

Matches graph (b).

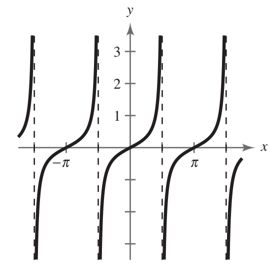
7. $y = \frac{1}{3}\tan x$

Period: π

Two consecutive asymptotes:

$x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	$-\frac{1}{3}$	0	$\frac{1}{3}$



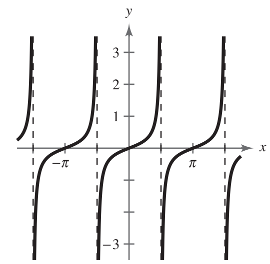
8. $y = \frac{1}{4}\tan x$

Period: π

Two consecutive asymptotes:

$x = -\frac{\pi}{2}, x = \frac{\pi}{2}$

x	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$
y	$-\frac{1}{4}$	0	$\frac{1}{4}$



9. $y = \tan 3x$

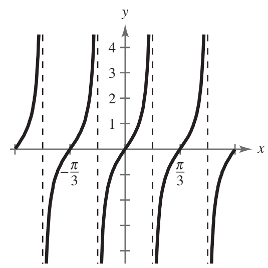
Period: $\frac{\pi}{3}$

Two consecutive asymptotes:

$3x = -\frac{\pi}{2} \Rightarrow x = -\frac{\pi}{6}$

$3x - \frac{\pi}{2} \Rightarrow x = \frac{\pi}{6}$

x	$-\frac{\pi}{12}$	0	$\frac{\pi}{12}$
y	-1	0	1



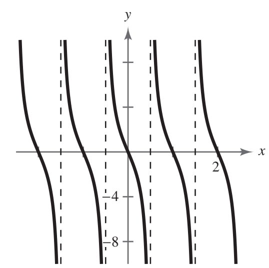
10. $y = -3\tan \pi x$

Period: $\frac{\pi}{\pi} = 1$

Two consecutive asymptotes:

$x = -\frac{1}{2}, x = \frac{1}{2}$

x	$-\frac{1}{4}$	0	$\frac{1}{4}$
y	3	0	-3



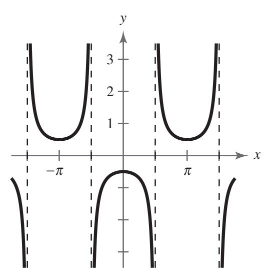
11. $y = -\frac{1}{2}\sec x$

Period: 2π

Two consecutive asymptotes:

$x = -\frac{\pi}{2}, x = \frac{\pi}{2}$

x	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$
y	-1	$-\frac{1}{2}$	-1



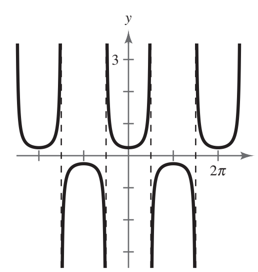
12. $y = \frac{1}{4}\sec x$

Period: 2π

Two consecutive asymptotes:

$x = -\frac{\pi}{2}, x = \frac{\pi}{2}$

x	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$
y	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$



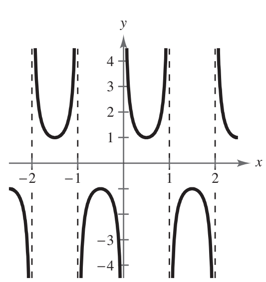
13. $y = \csc \pi x$

Period: $\frac{2\pi}{\pi} = 2$

Two consecutive asymptotes:

$x = 0, x = 1$

x	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{5}{6}$
y	2	1	2



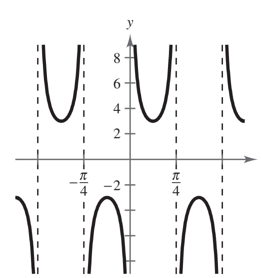
14. $y = 3\csc 4x$

Period: $\frac{2\pi}{4} = \frac{\pi}{2}$

Two consecutive asymptotes:

$x = 0, x = \frac{\pi}{4}$

x	$\frac{\pi}{24}$	$\frac{\pi}{8}$	$\frac{5\pi}{24}$
y	6	3	6



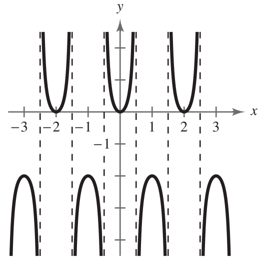
15. $y = \sec \pi x - 1$

Period: $\dfrac{2\pi}{\pi} = 2$

Two consecutive asymptotes:

$x = -\dfrac{1}{2}, x = \dfrac{1}{2}$

x	$-\dfrac{1}{3}$	0	$\dfrac{1}{3}$
y	1	0	1



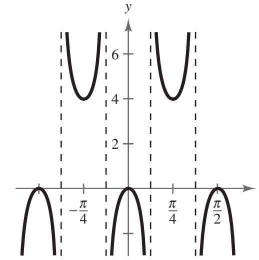
16. $y = -2 \sec 4x + 2$

Period: $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$

Two consecutive asymptotes:

$x = -\dfrac{\pi}{8}, x = \dfrac{\pi}{8}$

x	$-\dfrac{\pi}{12}$	0	$\dfrac{\pi}{12}$
y	-2	0	-2



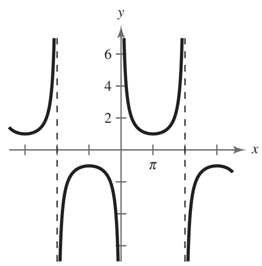
17. $y = \csc \dfrac{x}{2}$

Period: $\dfrac{2\pi}{1/2} = 4\pi$

Two consecutive asymptotes:

$x = 0, x = 2\pi$

x	$\dfrac{\pi}{3}$	π	$\dfrac{5\pi}{3}$
y	2	1	2



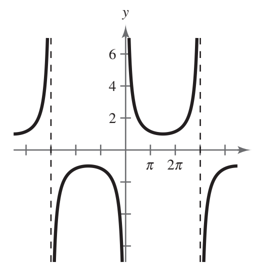
18. $y = \csc \dfrac{x}{3}$

Period: $\dfrac{2\pi}{1/3} = 6\pi$

Two consecutive asymptotes:

$x = 0, x = 3\pi$

x	$\dfrac{\pi}{2}$	$\dfrac{3\pi}{2}$	$\dfrac{5\pi}{2}$
y	2	1	2



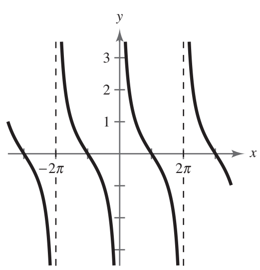
19. $y = \cot \dfrac{x}{2}$

Period: $\dfrac{\pi}{1/2} = 2\pi$

Two consecutive asymptotes:

$\dfrac{x}{2} = 0 \Rightarrow x = 0$

$\dfrac{x}{2} = \pi \Rightarrow x = 2\pi$



x	$\dfrac{\pi}{2}$	π	$\dfrac{3\pi}{2}$
y	1	0	-1

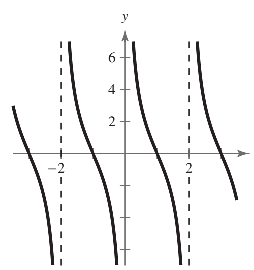
20. $y = 3 \cot \dfrac{\pi x}{2}$

Period: $\dfrac{\pi}{\pi/2} = 2$

Two consecutive asymptotes:

$x = 0, x = 2$

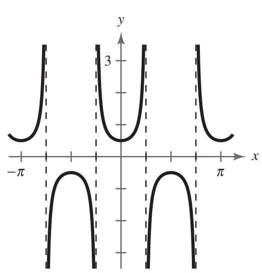
x	$\dfrac{1}{2}$	1	$\dfrac{3}{2}$
y	3	0	-3



21. $y = \dfrac{1}{2} \sec 2x$

Period: $\dfrac{2\pi}{2} = \pi$

x	$-\dfrac{\pi}{6}$	0	$\dfrac{\pi}{6}$
y	1	$\dfrac{1}{2}$	1



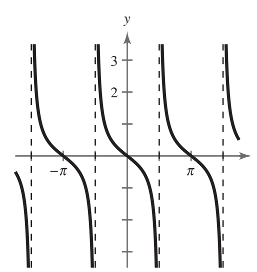
22. $y = -\dfrac{1}{2} \tan x$

Period: π

Two consecutive asymptotes:

$x = -\dfrac{\pi}{2}, x = \dfrac{\pi}{2}$

x	$-\dfrac{\pi}{4}$	0	$\dfrac{\pi}{4}$
y	$\dfrac{1}{2}$	0	$-\dfrac{1}{2}$



23. $y = \tan \dfrac{\pi x}{4}$

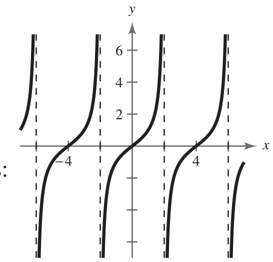
Period: $\dfrac{\pi}{\pi/4} = 4$

Two consecutive asymptotes:

$\dfrac{\pi x}{4} = -\dfrac{\pi}{2} \Rightarrow x = -2$

$\dfrac{\pi x}{4} = \dfrac{\pi}{2} \Rightarrow x = 2$

x	-1	0	1
y	-1	0	1



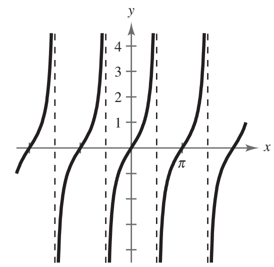
24. $y = \tan(x + \pi)$

Period: π

Two consecutive asymptotes:

$x = -\dfrac{\pi}{2}, x = \dfrac{\pi}{2}$

x	$-\dfrac{\pi}{4}$	0	$\dfrac{\pi}{4}$
y	-1	0	1



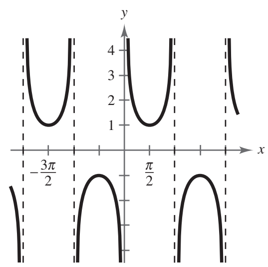
25. $y = \csc(\pi - x)$

Period: 2π

Two consecutive asymptotes:

$x = 0, x = \pi$

x	$\dfrac{\pi}{6}$	$\dfrac{\pi}{2}$	$\dfrac{5\pi}{6}$
y	2	1	2



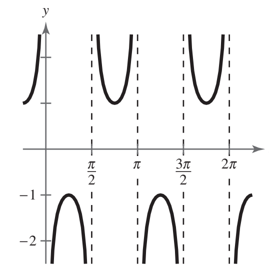
26. $y = \csc(2x - \pi)$

Period: $\dfrac{2\pi}{2} = \pi$

Two consecutive asymptotes:

$x = 0, x = \dfrac{\pi}{2}$

x	$\dfrac{\pi}{12}$	$\dfrac{\pi}{4}$	$\dfrac{5\pi}{12}$
y	-2	-1	-2



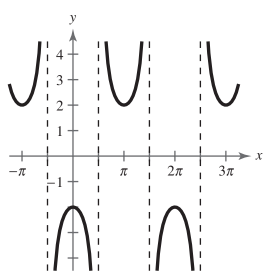
27. $y = 2\sec(x + \pi)$

Period: 2π

Two consecutive asymptotes:

$x = -\dfrac{\pi}{2}, x = \dfrac{\pi}{2}$

x	$-\dfrac{\pi}{3}$	0	$\dfrac{\pi}{3}$
y	-4	-2	-4



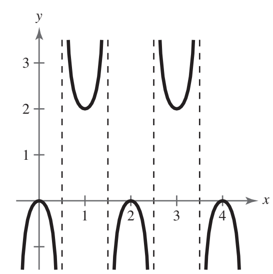
28. $y = -\sec \pi x + 1$

Period: $\dfrac{2\pi}{\pi} = 2$

Two consecutive asymptotes:

$x = -\dfrac{1}{2}, x = \dfrac{1}{2}$

x	$-\dfrac{1}{3}$	0	$\dfrac{1}{3}$
y	-1	0	1



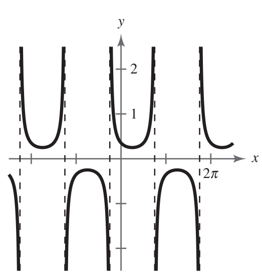
29. $y = \dfrac{1}{4}\csc\left(x + \dfrac{\pi}{4}\right)$

Period: 2π

Two consecutive asymptotes:

$x = -\dfrac{\pi}{4}, x = \dfrac{3\pi}{4}$

x	$-\dfrac{\pi}{12}$	$\dfrac{\pi}{4}$	$\dfrac{7\pi}{12}$
y	$\dfrac{1}{2}$	$\dfrac{1}{4}$	$\dfrac{1}{2}$



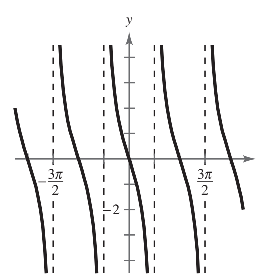
30. $y = 2\cot\left(x + \dfrac{\pi}{2}\right)$

Period: π

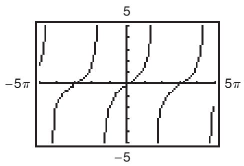
Two consecutive asymptotes:

$x = -\dfrac{\pi}{2}, x = \dfrac{\pi}{2}$

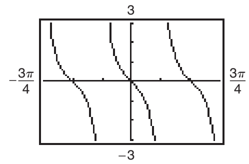
x	$-\dfrac{\pi}{4}$	0	$\dfrac{\pi}{4}$
y	2	0	-2



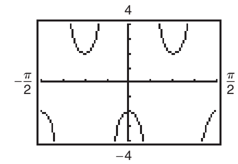
31. $y = \tan \dfrac{x}{3}$



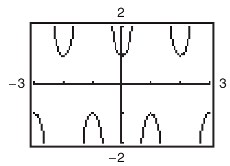
32. $y = -\tan 2x$



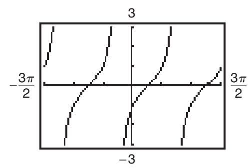
33. $y = -2 \sec 4x = \dfrac{-2}{\cos 4x}$



34. $y = \sec \pi x \implies y = \dfrac{1}{\cos(\pi x)}$

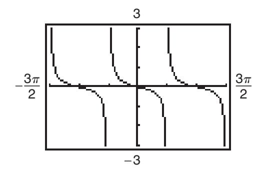


35. $y = \tan\left(x - \dfrac{\pi}{4}\right)$



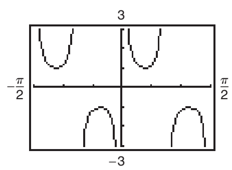
36. $y = \dfrac{1}{4}\cot\left(x - \dfrac{\pi}{2}\right)$

$= \dfrac{1}{4\tan\left(x - \dfrac{\pi}{2}\right)}$



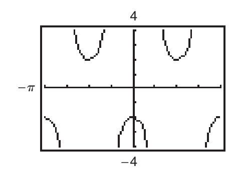
37. $y = -\csc(4x - \pi)$

$y = \dfrac{-1}{\sin(4x - \pi)}$

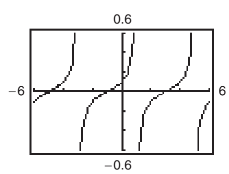


38. $y = 2\sec(2x - \pi) \implies$

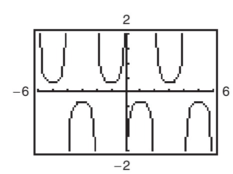
$y = \dfrac{2}{\cos(2x - \pi)}$



39. $y = 0.1 \tan\left(\dfrac{\pi x}{4} + \dfrac{\pi}{4}\right)$

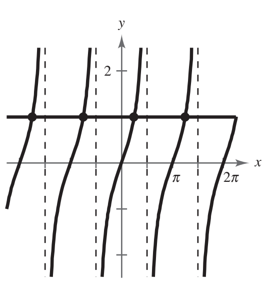


40. $y = \dfrac{1}{3}\sec\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right) \implies y = \dfrac{1}{3\cos\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right)}$



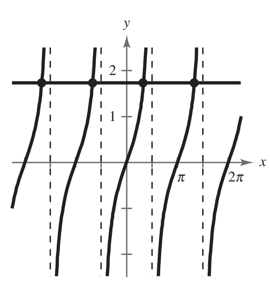
41. $\tan x = 1$

$x = -\dfrac{7\pi}{4}, -\dfrac{3\pi}{4}, \dfrac{\pi}{4}, \dfrac{5\pi}{4}$



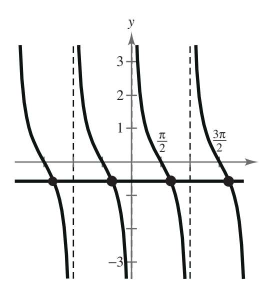
42. $\tan x = \sqrt{3}$

$x = -\dfrac{5\pi}{3}, -\dfrac{2\pi}{3}, \dfrac{\pi}{3}, \dfrac{4\pi}{3}$



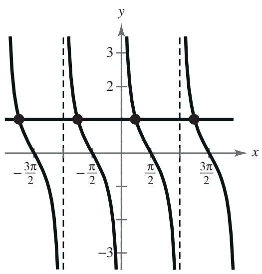
43. $\cot x = -\dfrac{\sqrt{3}}{3}$

$x = -\dfrac{4\pi}{3}, -\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{5\pi}{3}$



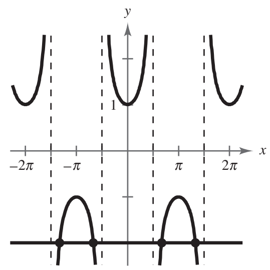
44. $\cot x = 1$

$$x = -\frac{7\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}, \frac{5\pi}{4}$$



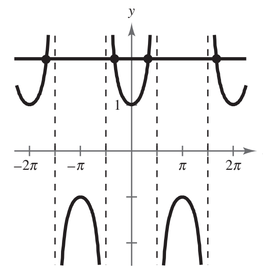
45. $\sec x = -2$

$$x = \pm\frac{2\pi}{3}, \pm\frac{4\pi}{3}$$



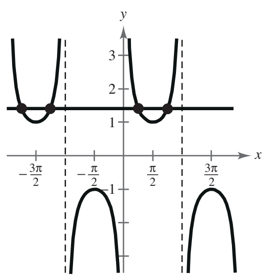
46. $\sec x = 2$

$$x = -\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$$



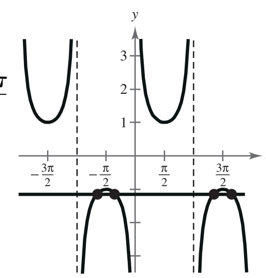
47. $\csc x = \sqrt{2}$

$$x = -\frac{7\pi}{4}, -\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$$



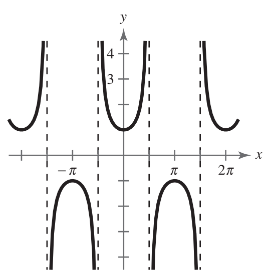
48. $\csc x = -\frac{2\sqrt{3}}{3}$

$$x = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$



49. $f(x) = \sec x = \dfrac{1}{\cos x}$

$$f(-x) = \sec(-x) = \frac{1}{\cos(-x)} = \frac{1}{\cos x} = f(x)$$

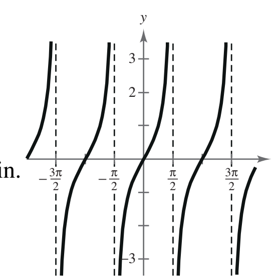


Thus, $f(x) = \sec x$ is an even function and the graph has y-axis symmetry.

50. $f(x) = \tan x$

$\tan(-x) = -\tan x$

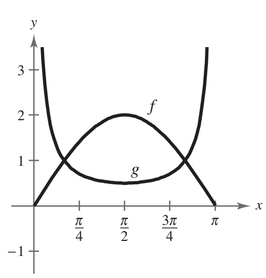
Thus, the function is odd and the graph of $y = \tan x$ is symmetric about the origin.



51. $f(x) = 2 \sin x$

$g(x) = \dfrac{1}{2} \csc x$

(a)

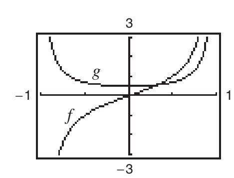


(b) $f > g$ on the interval, $\dfrac{\pi}{6} < x < \dfrac{5\pi}{6}$

(c) As $x \to \pi$, $f(x) = 2 \sin x \to 0$ and $g(x) = \frac{1}{2} \csc x \to \pm\infty$ since $g(x)$ is the reciprocal of $f(x)$.

52. $f(x) = \tan \dfrac{\pi x}{2}$, $g(x) = \dfrac{1}{2} \sec \dfrac{\pi x}{2}$

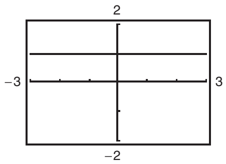
(a)



(b) The interval in which $f < g$ is $\left(-1, \frac{1}{3}\right)$.

(c) The interval in which $2f < 2g$ is $\left(-1, \frac{1}{3}\right)$, which is the same interval as part (b).

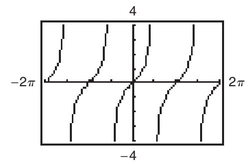
53. $y_1 = \sin x \csc x$ and $y_2 = 1$



$\sin x \csc x = \sin x\left(\dfrac{1}{\sin x}\right) = 1, \sin x \neq 0$

The expressions are equivalent except when $\sin x = 0$ and y_1 is undefined.

54. $y_1 = \sin x \sec x, y_2 = \tan x$



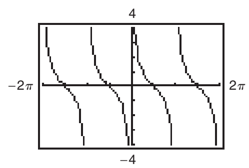
$\sin x \sec x = \sin x \dfrac{1}{\cos x} = \dfrac{\sin x}{\cos x} = \tan x$

The expressions are equivalent.

55. $y_1 = \dfrac{\cos x}{\sin x}$ and $y_2 = \cot x = \dfrac{1}{\tan x}$

$\cot x = \dfrac{\cos x}{\sin x}$

The expressions are equivalent.

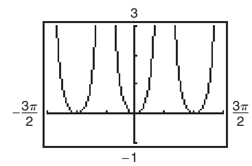


56. $y_1 = \sec^2 x - 1, y_2 = \tan^2 x$

$1 + \tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

The expressions are equivalent.



57. $f(x) = |x \cos x|$

As $x \longrightarrow 0, f(x) \longrightarrow 0$ and $f(x) > 0$.

Matches graph (d).

58. $f(x) = x \sin x$

Matches graph (a) as $x \to 0,\ f(x) \to 0$.

59. $g(x) = |x| \sin x$

As $x \longrightarrow 0, g(x) \longrightarrow 0$ and $g(x)$ is odd.

Matches graph (b).

60. $g(x) = |x| \cos x$

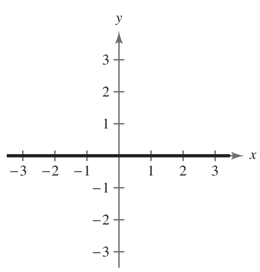
Matches graph (c) as $x \to 0,\ g(x) \to 0$.

61. $f(x) = \sin x + \cos\left(x + \dfrac{\pi}{2}\right)$

$g(x) = 0$

$f(x) = g(x)$

The graph is the line $y = 0$.

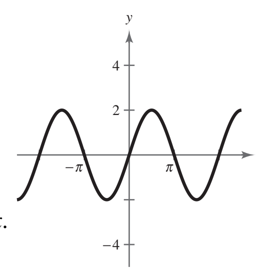


62. $f(x) = \sin x - \cos\left(x + \dfrac{\pi}{2}\right)$

$g(x) = 2 \sin x$

It appears that $f(x) = g(x)$. That is,

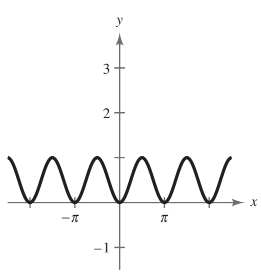
$\sin x - \cos\left(x + \dfrac{\pi}{2}\right) = 2 \sin x.$



63. $f(x) = \sin^2 x$

$g(x) = \dfrac{1}{2}(1 - \cos 2x)$

$f(x) = g(x)$

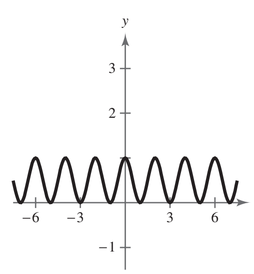


64. $f(x) = \cos^2 \dfrac{\pi x}{2}$

$g(x) = \dfrac{1}{2}(1 + \cos \pi x)$

It appears that $f(x) = g(x)$. That is,

$\cos^2 \dfrac{\pi x}{2} = \dfrac{1}{2}(1 + \cos \pi x).$

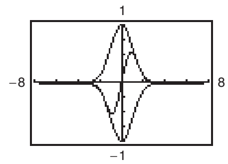


65. $g(x) = e^{-x^2/2} \sin x$

$-e^{-x^2/2} \le g(x) \le e^{-x^2/2}$

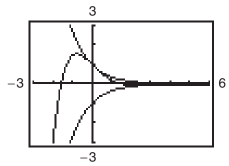
The damping factor is $y = e^{-x^2/2}$.

As $x \to \infty$, $g(x) \to 0$.



66. $f(x) = e^{-x} \cos x$

Damping factor: e^{-x}

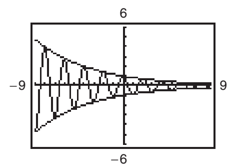


As $x \to \infty$, $f(x) \to 0$.

67. $f(x) = 2^{-x/4} \cos \pi x$

$-2^{-x/4} \le f(x) \le 2^{-x/4}$

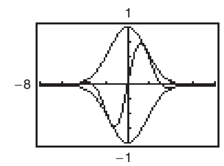
Damping factor: $y = 2^{-x/4}$.



As $x \to \infty$, $f(x) \to 0$.

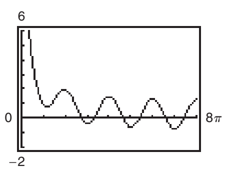
68. $h(x) = 2^{-x^2/4} \sin x$

Damping factor: $2^{-x^2/4}$



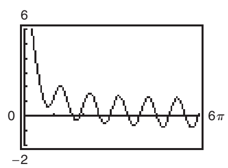
As $x \to \infty$, $h(x) \to 0$.

69. $y = \dfrac{6}{x} + \cos x,\ x > 0$



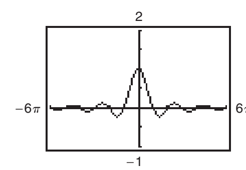
As $x \to 0$, $y \to \infty$.

70. $y = \dfrac{4}{x} + \sin 2x, x > 0$



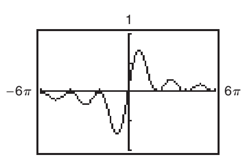
As $x \to 0$, $y \to \infty$.

71. $g(x) = \dfrac{\sin x}{x}$



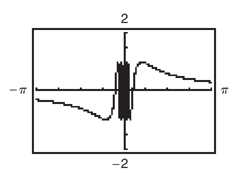
As $x \to 0$, $g(x) \to 1$.

72. $f(x) = \dfrac{1 - \cos x}{x}$



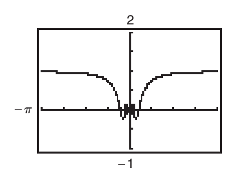
As $x \to 0$, $f(x) \to 0$.

73. $f(x) = \sin \dfrac{1}{x}$



As $x \to 0$, $f(x)$ oscillates between -1 and 1.

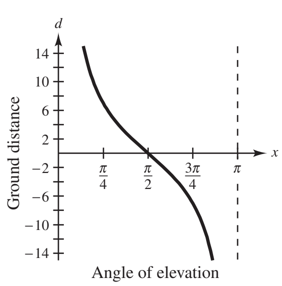
74. $h(x) = x \sin \dfrac{1}{x}$



As $x \to 0$, $h(x)$ oscillates.

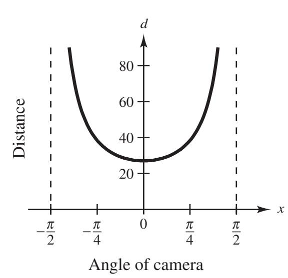
75. $\tan x = \dfrac{7}{d}$

$$d = \frac{7}{\tan x} = 7 \cot x$$

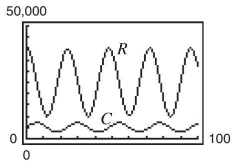


76. $\cos x = \dfrac{27}{d}$

$$d = \frac{27}{\cos x} = 27 \sec x, -\frac{\pi}{2} < x < \frac{\pi}{2}$$



77. $C = 5000 + 2000 \sin \dfrac{\pi t}{12}, R = 25{,}000 + 15{,}000 \cos \dfrac{\pi t}{12}$

(a) 

(b) As the predator population increases, the number of prey decreases. When the number of prey is small, the number of predators decreases.

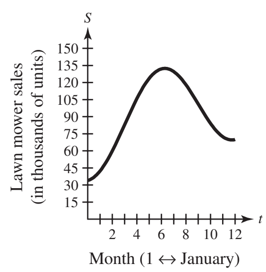
(c) The period for both C and R is:

$$p = \frac{2\pi}{\pi/12} = 24 \text{ months}$$

When the prey population is highest, the predator population is increasing most rapidly.
When the prey population is lowest, the predator population is decreasing most rapidly.
When the predator population is lowest, the prey population is increasing most rapidly.
When the predator population is highest, the prey population is decreasing most rapidly.

In addition, weather, food sources for the prey, hunting, all affect the populations of both the predator and the prey.

78. $S = 74 + 3t - 40 \cos \dfrac{\pi t}{6}$



79. $H(t) = 54.33 - 20.38 \cos \dfrac{\pi t}{6} - 15.69 \sin \dfrac{\pi t}{6}$

$L(t) = 39.36 - 15.70 \cos \dfrac{\pi t}{6} - 14.16 \sin \dfrac{\pi t}{6}$

(a) Period of $\cos \dfrac{\pi t}{6}$: $\dfrac{2\pi}{\pi/6} = 12$

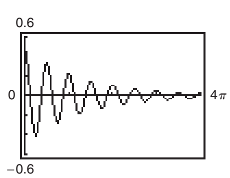
Period of $\sin \dfrac{\pi t}{6}$: $\dfrac{2\pi}{\pi/6} = 12$

Period of $H(t)$: 12 months

Period of $L(t)$: 12 months

(b) From the graph, it appears that the greatest difference between high and low temperatures occurs in summer. The smallest difference occurs in winter.

(c) The highest high and low temperatures appear to occur around the middle of July, roughly one month after the time when the sun is northernmost in the sky.

80. (a) 

$y = \dfrac{1}{2}e^{-t/4} \cos 4^t$

(b) The displacement is a damped sine wave.
$y \to 0$ as t increases.

81. True. Since

$$y = \csc x = \frac{1}{\sin x},$$

for a given value of x, the y-coordinate of $\csc x$ is the reciprocal of the y-coordinate of $\sin x$.

82. True.

$$y = \sec x = \frac{1}{\cos x}$$

If the reciprocal of $y = \sin x$ is translated $\pi/2$ units to the left, we have

$$y = \frac{1}{\sin\left(x + \frac{\pi}{2}\right)} = \frac{1}{\cos x} = \sec x.$$

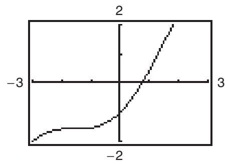
83. As $x \to \frac{\pi}{2}$ from the left, $f(x) = \tan x \to \infty$.

As $x \to \frac{\pi}{2}$ from the right, $f(x) = \tan x \to -\infty$.

84. As $x \to \pi$ from the left, $f(x) = \csc x \to \infty$.

As $x \to \pi$ from the right, $f(x) = \csc x \to -\infty$.

85. $f(x) = x - \cos x$

(a) 

The zero between 0 and 1 occurs at $x \approx 0.7391$.

(b) $x_n = \cos(x_{n-1})$

$x_0 = 1$

$x_1 = \cos 1 \approx 0.5403$

$x_2 = \cos 0.5403 \approx 0.8576$

$x_3 = \cos 0.8576 \approx 0.6543$

$x_4 = \cos 0.6543 \approx 0.7935$

$x_5 = \cos 0.7935 \approx 0.7014$

$x_6 = \cos 0.7014 \approx 0.7640$

$x_7 = \cos 0.7640 \approx 0.7221$

$x_8 = \cos 0.7221 \approx 0.7504$

$x_9 = \cos 0.7504 \approx 0.7314$

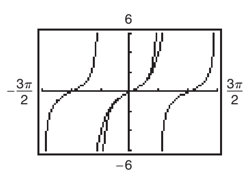
$\vdots$

This sequence appears to be approaching the zero of f: $x \approx 0.7391$.

86. $y = \tan x$

$$y = x + \frac{2x^3}{3!} + \frac{16x^5}{5!}$$

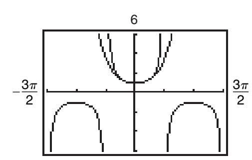
The graphs are nearly the same for $-1.1 < x < 1.1$.



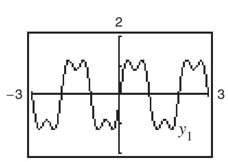
87. $y_1 = \sec x$

$$y_2 = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!}$$

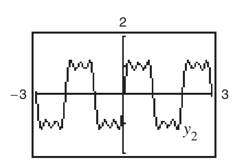
The graph appears to coincide on the interval $-1.1 \le x \le 1.1$.



88. (a) $y_1 = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x\right)$



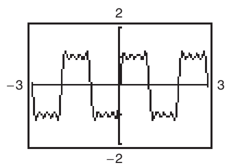
$y_2 = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x\right)$



—CONTINUED—

88. —CONTINUED—

(b) $y_3 = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x + \frac{1}{7}\sin 7\pi x\right)$



(c) $y_4 = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x + \frac{1}{7}\sin 7\pi x + \frac{1}{9}\sin 9\pi x\right)$

89. $e^{2x} = 54$

$$2x = \ln 54$$
$$x = \frac{\ln 54}{2} \approx 1.994$$

90. $8^{3x} = 98$

$$3x = \log_8 98$$
$$x = \frac{\ln 98}{3 \ln 8} \approx 0.735$$

91. $\frac{300}{1 + e^{-x}} = 100$

$$\frac{300}{100} = 1 + e^{-x}$$
$$3 = 1 + e^{-x}$$
$$2 = e^{-x}$$
$$\ln 2 = -x$$
$$x = -\ln 2 \approx -0.693$$

92. $\left(1 + \frac{0.15}{365}\right)^{365t} = 5$

$$1 + \frac{0.15}{365} \approx 1.00041096$$
$$1.00041096^{365t} = 5$$
$$365t = \log_{1.00041096} 5$$
$$t = \frac{1}{365}\left(\frac{\log_{10} 5}{\log_{10} 1.00041096}\right) \approx 10.732$$

93. $\ln(3x - 2) = 73$

$$3x - 2 = e^{73}$$
$$3x = 2 + e^{73}$$
$$x = \frac{2 + e^{73}}{3}$$
$$\approx 1.684 \times 10^{31}$$

94. $\ln(14 - 2x) = 68$

$$14 - 2x = e^{68}$$
$$14 - e^{68} = 2x$$
$$x = \frac{14 - e^{68}}{2} \approx -1.702 \times 10^{29}$$

95. $\ln(x^2 + 1) = 3.2$

$$x^2 + 1 = e^{3.2}$$
$$x^2 = e^{3.2} - 1$$
$$x = \pm\sqrt{e^{3.2} - 1} \approx \pm 4.851$$

96. $\ln\sqrt{x + 4} = 5$

$$\tfrac{1}{2}\ln(x + 4) = 5$$
$$\ln(x + 4) = 10$$
$$x + 4 = e^{10}$$
$$x = e^{10} - 4$$
$$\approx 22{,}022.466$$

97. $\log_8 x + \log_8(x - 1) = \frac{1}{3}$

$$\log_8[x(x - 1)] = \tfrac{1}{3}$$
$$x(x - 1) = 8^{1/3}$$
$$x^2 - x = 2$$
$$x^2 - x - 2 = 0$$
$$(x - 2)(x + 1) = 0$$
$$x = 2, -1$$

$x = -1$ is extraneous (not in the domain of $\log_8 x$) so only $x = 2$ is a solution.

98. $\log_6 x + \log_6(x^2 - 1) = \log_6(64x)$

$$\log_6(x(x^2 - 1)) = \log_6(64x)$$
$$x(x^2 - 1) = 64x$$
$$x^2 - 1 = 64$$
$$x = \pm\sqrt{65}$$

Since $-\sqrt{65}$ is not in the domain of $\log_6 x$, the only solution is $x = \sqrt{65} \approx 8.062$.

Section 4.7 Inverse Trigonometric Functions

- You should know the definitions, domains, and ranges of $y = \arcsin x$, $y = \arccos x$, and $y = \arctan x$.

Function	*Domain*	*Range*
$y = \arcsin x \Rightarrow x = \sin y$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \arccos x \Rightarrow x = \cos y$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \arctan x \Rightarrow x = \tan y$	$-\infty < x < \infty$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$

- You should know the inverse properties of the inverse trigonometric functions.

$\sin(\arcsin x) = x$ and $\arcsin(\sin y) = y, -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\cos(\arccos x) = x$ and $\arccos(\cos y) = y, 0 \le y \le \pi$

$\tan(\arctan x) = x$ and $\arctan(\tan y) = y, -\frac{\pi}{2} < y < \frac{\pi}{2}$

- You should be able to use the triangle technique to convert trigonometric functions of inverse trigonometric functions into algebraic expressions.

Vocabulary Check

	Function	*Alternative Notation*	*Domain*	*Range*
1.	$y = \arcsin x$	$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
2.	$y = \arccos x$	$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
3.	$y = \arctan x$	$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

1. $y = \arcsin \frac{1}{2} \Rightarrow \sin y = \frac{1}{2}$ for $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = \frac{\pi}{6}$

2. $y = \arcsin 0 \Rightarrow \sin y = 0$ for $-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = 0$

3. $y = \arccos \frac{1}{2} \Rightarrow \cos y = \frac{1}{2}$ for $0 \le y \le \pi \Rightarrow y = \frac{\pi}{3}$

4. $y = \arccos 0 \Rightarrow \cos y = 0$ for $0 \le y \le \pi \Rightarrow y = \frac{\pi}{2}$

5. $y = \arctan \frac{\sqrt{3}}{3} \Rightarrow \tan y = \frac{\sqrt{3}}{3}$ for

$-\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = \frac{\pi}{6}$

6. $y = \arctan(-1) \Rightarrow \tan y = -1$ for

$-\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = -\frac{\pi}{4}$

7. $y = \arccos\left(-\frac{\sqrt{3}}{2}\right) \Rightarrow \cos y = -\frac{\sqrt{3}}{2}$ for

$0 \le y \le \pi \Rightarrow y = \frac{5\pi}{6}$

8. $y = \arcsin\left(-\frac{\sqrt{2}}{2}\right) \Rightarrow \sin y = -\frac{\sqrt{2}}{2}$ for

$-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = -\frac{\pi}{4}$

9. $y = \arctan\left(-\sqrt{3}\right) \Rightarrow \tan y = -\sqrt{3}$ for

$-\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = -\frac{\pi}{3}$

10. $y = \arctan\left(\sqrt{3}\right) \Rightarrow \tan y = \sqrt{3}$ for

$-\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = \frac{\pi}{3}$

11. $y = \arccos\left(-\frac{1}{2}\right) \Rightarrow \cos y = -\frac{1}{2}$ for

$0 \le y \le \pi \Rightarrow y = \frac{2\pi}{3}$

12. $y = \arcsin\frac{\sqrt{2}}{2} \Rightarrow \sin y = \frac{\sqrt{2}}{2}$ for

$-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = \frac{\pi}{4}$

13. $y = \arcsin\frac{\sqrt{3}}{2} \Rightarrow \sin y = \frac{\sqrt{3}}{2}$ for

$-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Rightarrow y = \frac{\pi}{3}$

14. $y = \arctan\left(-\frac{\sqrt{3}}{3}\right) \Rightarrow \tan y = -\frac{\sqrt{3}}{3}$ for

$-\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = -\frac{\pi}{6}$

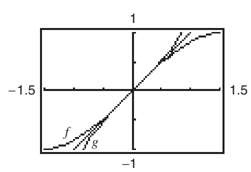
15. $y = \arctan 0 \Rightarrow \tan y = 0$ for $-\frac{\pi}{2} < y < \frac{\pi}{2} \Rightarrow y = 0$

16. $y = \arccos 1 \Rightarrow \cos y = 1$ for $0 \le y \le \pi \Rightarrow y = 0$

17. $f(x) = \sin x$

$g(x) = \arcsin x$

$y = x$

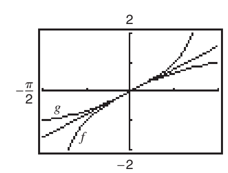


18. $f(x) = \tan x$ and $g(x) = \arctan x$

Graph $y_1 = \tan x$.

Graph $y_2 = \tan^{-1} x$.

Graph $y_3 = x$.



19. $\arccos 0.28 = \cos^{-1} 0.28 \approx 1.29$

20. $\arcsin 0.45 \approx 0.47$

21. $\arcsin(-0.75) = \sin^{-1}(-0.75) \approx -0.85$

22. $\arccos(-0.7) \approx 2.35$

23. $\arctan(-3) = \tan^{-1}(-3) \approx -1.25$

24. $\arctan 15 \approx 1.50$

25. $\arcsin 0.31 = \sin^{-1} 0.31 \approx 0.32$

26. $\arccos 0.26 \approx 1.31$

27. $\arccos(-0.41) = \cos^{-1}(-0.41) \approx 1.99$

28. $\arcsin(-0.125) \approx -0.13$

29. $\arctan 0.92 = \tan^{-1} 0.92 \approx 0.74$

30. $\arctan 2.8 \approx 1.23$

31. $\arcsin\left(\frac{3}{4}\right) = \sin^{-1}(0.75) \approx 0.85$

32. $\arccos\left(-\frac{1}{3}\right) \approx 1.91$

33. $\arctan\left(\frac{7}{2}\right) = \tan^{-1}(3.5) \approx 1.29$

34. $\arctan\left(-\frac{95}{7}\right) \approx -1.50$

35. This is the graph of $y = \arctan x$. The coordinates are $\left(-\sqrt{3}, -\frac{\pi}{3}\right)$, $\left(-\frac{\sqrt{3}}{3}, -\frac{\pi}{6}\right)$, and $\left(1, \frac{\pi}{4}\right)$.

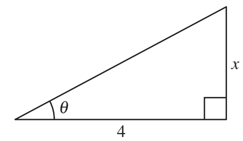
36. $\arccos(-1) = \pi$

$$\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

37. $\tan\theta = \dfrac{x}{4}$

$$\theta = \arctan\frac{x}{4}$$

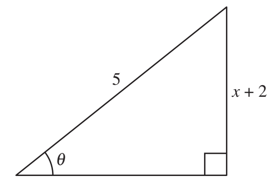


38. $\cos\theta = \dfrac{4}{x}$

$$\theta = \arccos\frac{4}{x}$$

39. $\sin\theta = \dfrac{x+2}{5}$

$$\theta = \arcsin\left(\frac{x+2}{5}\right)$$

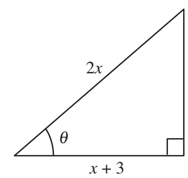


40. $\tan\theta = \dfrac{x+1}{10}$

$$\theta = \arctan\left(\frac{x+1}{10}\right)$$

41. $\cos\theta = \dfrac{x+3}{2x}$

$$\theta = \arccos\left(\frac{x+3}{2x}\right)$$



42. $\tan\theta = \dfrac{x-1}{x^2-1} = \dfrac{1}{x+1}$

$$\theta = \arctan\frac{1}{x+1}$$

$x \neq 1$

43. $\sin(\arcsin 0.3) = 0.3$

44. $\tan(\arctan 25) = 25$

45. $\cos[\arccos(-0.1)] = -0.1$

46. $\sin[\arcsin(-0.2)] = -0.2$

47. $\arcsin(\sin 3\pi) = \arcsin(0) = 0$

Note: 3π is not in the range of the arcsine function.

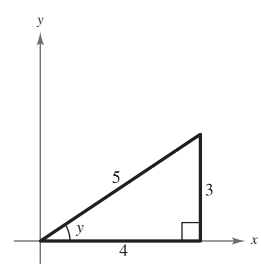
48. $\arccos\left(\cos\dfrac{7\pi}{2}\right) = \arccos 0 = \dfrac{\pi}{2}$

Note: $\dfrac{7\pi}{2}$ is not in the range of the arccosine function.

49. Let $y = \arctan\dfrac{3}{4}$,

$\tan y = \dfrac{3}{4},\ 0 < y < \dfrac{\pi}{2}$,

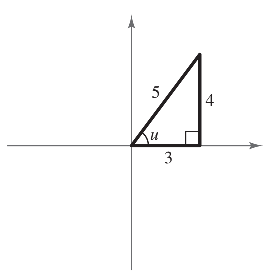
and $\sin y = \dfrac{3}{5}$.



50. Let $u = \arcsin\dfrac{4}{5}$,

$\sin u = \dfrac{4}{5}, 0 < u < \dfrac{\pi}{2}$,

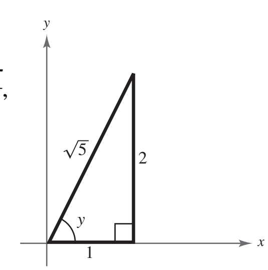
$\sec\left(\arcsin\dfrac{4}{5}\right) = \sec u = \dfrac{5}{3}$.



51. Let $y = \arctan 2$,

$\tan y = 2 = \dfrac{2}{1},\ 0 < y < \dfrac{\pi}{2}$,

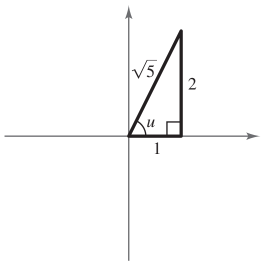
and $\cos y = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$.



52. Let $u = \arccos \dfrac{\sqrt{5}}{5}$,

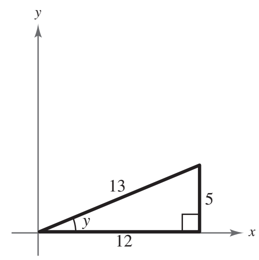
$\cos u = \dfrac{\sqrt{5}}{5}$, $0 < u < \dfrac{\pi}{2}$,

$\sin\left(\arccos \dfrac{\sqrt{5}}{5}\right) = \sin u = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$.



53. Let $y = \arcsin \dfrac{5}{13}$,

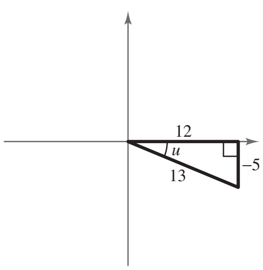
$\sin y = \dfrac{5}{13}$, $0 < y < \dfrac{\pi}{2}$, and $\cos y = \dfrac{12}{13}$.



54. Let $u = \arctan\left(-\dfrac{5}{12}\right)$,

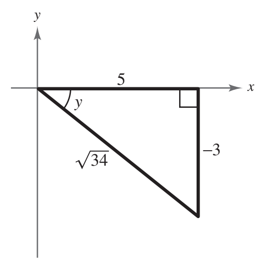
$\tan u = -\dfrac{5}{12}$, $-\dfrac{\pi}{2} < u < 0$,

$\csc\left[\arctan\left(-\dfrac{5}{12}\right)\right] = \csc u = -\dfrac{13}{5}$.



55. Let $y = \arctan\left(-\dfrac{3}{5}\right)$,

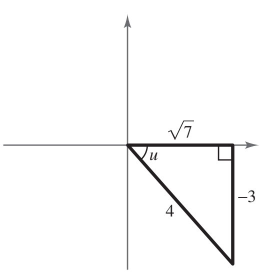
$\tan y = -\dfrac{3}{5}$, $-\dfrac{\pi}{2} < y < 0$, and $\sec y = \dfrac{\sqrt{34}}{5}$.



56. Let $u = \arcsin\left(-\dfrac{3}{4}\right)$,

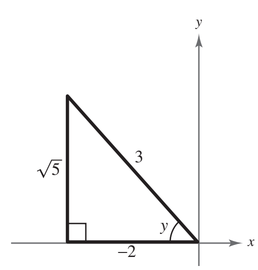
$\sin u = -\dfrac{3}{4}$, $-\dfrac{\pi}{2} < u < 0$,

$\tan\left[\arcsin\left(-\dfrac{3}{4}\right)\right] = \tan u = -\dfrac{3}{\sqrt{7}} = -\dfrac{3\sqrt{7}}{7}$.



57. Let $y = \arccos\left(-\dfrac{2}{3}\right)$,

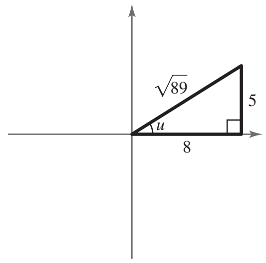
$\cos y = -\dfrac{2}{3}$, $\dfrac{\pi}{2} < y < \pi$, and $\sin y = \dfrac{\sqrt{5}}{3}$.



58. Let $u = \arctan \frac{5}{8}$,

$\tan u = \frac{5}{8}, 0 < u < \frac{\pi}{2}$,

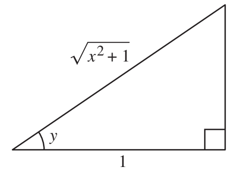
$\cot\left(\arctan \frac{5}{8}\right) = \cot u = \frac{8}{5}$.



59. Let $y = \arctan x$,

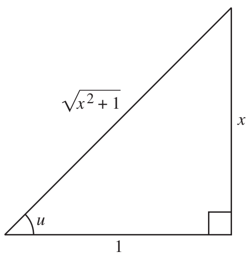
$\tan y = x = \frac{x}{1}$,

and $\cot y = \frac{1}{x}$.



60. Let $u = \arctan x$, $\tan u = x = \frac{x}{1}$,

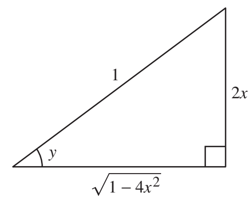
$\sin(\arctan x) = \sin u = \frac{x}{\sqrt{x^2 + 1}}$.



61. Let $y = \arcsin(2x)$,

$\sin y = 2x = \frac{2x}{1}$,

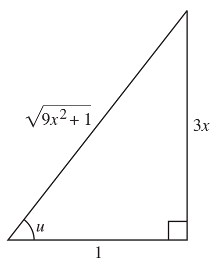
and $\cos y = \sqrt{1 - 4x^2}$.



62. Let $u = \arctan 3x$,

$\tan u = 3x = \frac{3x}{1}$,

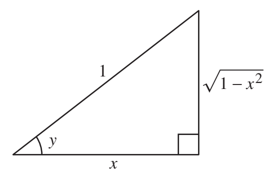
$\sec(\arctan 3x) = \sec u = \sqrt{9x^2 + 1}$.



63. Let $y = \arccos x$,

$\cos y = x = \frac{x}{1}$,

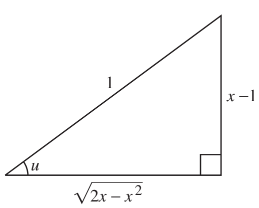
and $\sin y = \sqrt{1 - x^2}$.



64. Let $u = \arcsin(x - 1)$,

$\sin u = x - 1 = \frac{x - 1}{1}$,

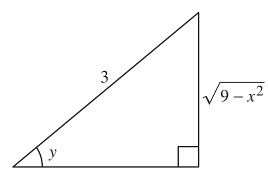
$\sec[\arcsin(x - 1)] = \sec u = \frac{1}{\sqrt{2x - x^2}}$.



65. Let $y = \arccos\left(\frac{x}{3}\right)$,

$\cos y = \frac{x}{3}$,

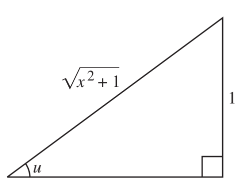
and $\tan y = \frac{\sqrt{9 - x^2}}{x}$.



66. Let $u = \arctan \frac{1}{x}$,

$\tan u = \frac{1}{x}$,

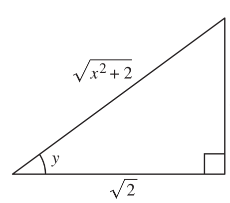
$\cot\left(\arctan \frac{1}{x}\right) = \cot u = x$.



67. Let $y = \arctan \frac{x}{\sqrt{2}}$,

$\tan y = \frac{x}{\sqrt{2}}$,

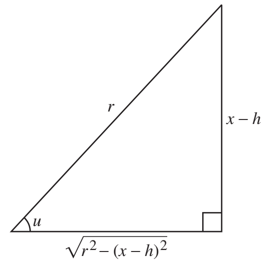
and $\csc y = \frac{\sqrt{x^2 + 2}}{x}$.



68. Let $u = \arcsin \dfrac{x - h}{r}$,

$\sin u = \dfrac{x - h}{r}$,

$\cos\left(\arcsin \dfrac{x - h}{r}\right) = \cos u = \dfrac{\sqrt{r^2 - (x - h)^2}}{r}$.



69. $f(x) = \sin(\arctan 2x)$, $g(x) = \dfrac{2x}{\sqrt{1 + 4x^2}}$

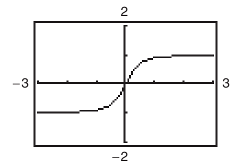
They are equal. Let $y = \arctan 2x$,

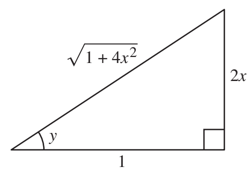
$\tan y = 2x = \dfrac{2x}{1}$,

and $\sin y = \dfrac{2x}{\sqrt{1 + 4x^2}}$.

$g(x) = \dfrac{2x}{\sqrt{1 + 4x^2}} = f(x)$

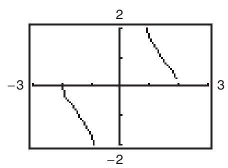
The graph has horizontal asymptotes at $y = \pm 1$.





70. $f(x) = \tan\left(\arccos \dfrac{x}{2}\right)$

$g(x) = \dfrac{\sqrt{4 - x^2}}{x}$



Asymptote: $x = 0$

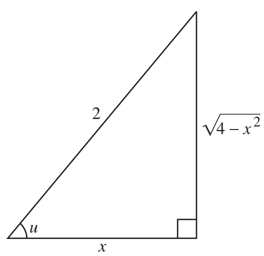
These are equal because:

Let $u = \arccos \dfrac{x}{2}$.

$f(x) = \tan\left(\arccos \dfrac{x}{2}\right) = \tan u$

$= \dfrac{\sqrt{4 - x^2}}{x} = g(x)$

Thus, $f(x) = g(x)$.



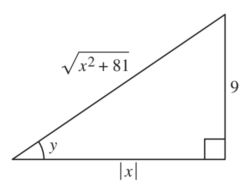
71. Let $y = \arctan \dfrac{9}{x}$.

$\tan y = \dfrac{9}{x}$ and $\sin y = \dfrac{9}{\sqrt{x^2 + 81}}, x > 0; \dfrac{-9}{\sqrt{x^2 + 81}}, x < 0.$

Thus,

$\arcsin y = \dfrac{9}{\sqrt{x^2 + 81}}, x > 0;$

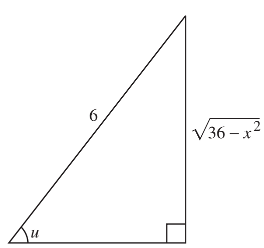
$\arcsin y = \dfrac{-9}{\sqrt{x^2 + 81}}, x < 0.$



72. If $\arcsin \dfrac{\sqrt{36 - x^2}}{6} = u$,

then $\sin u = \dfrac{\sqrt{36 - x^2}}{6}$,

$\arcsin \dfrac{\sqrt{36 - x^2}}{6} = \arccos \dfrac{x}{6}$.

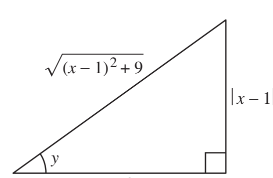


73. Let $y = \arccos \dfrac{3}{\sqrt{x^2 - 2x + 10}}$. Then,

$\cos y = \dfrac{3}{\sqrt{x^2 - 2x + 10}} = \dfrac{3}{\sqrt{(x - 1)^2 + 9}}$

and $\sin y = \dfrac{|x - 1|}{\sqrt{(x - 1)^2 + 9}}$.

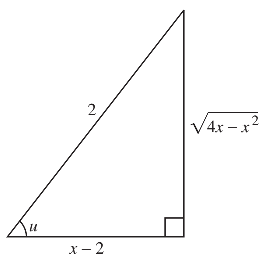
Thus, $y = \arcsin \dfrac{|x - 1|}{\sqrt{x^2 - 2x + 10}}$.



74. If $\arccos \dfrac{x-2}{2} = u$,

then $\cos u = \dfrac{x-2}{2}$,

$\arccos \dfrac{x-2}{2} = \arctan \dfrac{\sqrt{4x - x^2}}{x-2}$.

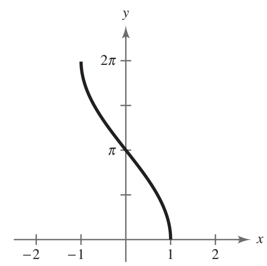


75. $y = 2 \arccos x$

Domain: $-1 \le x \le 1$

Range: $0 \le y \le 2\pi$

This is the graph of $f(x) = \arccos x$ with a factor of 2.

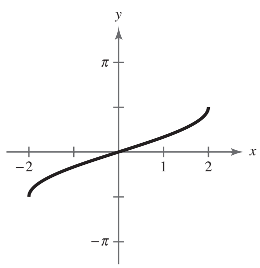


76. $y = \arcsin \dfrac{x}{2}$

Domain: $-2 \le x \le 2$

Range: $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$

This is the graph of $f(x) = \arcsin x$ with a horizontal stretch of a factor of 2.

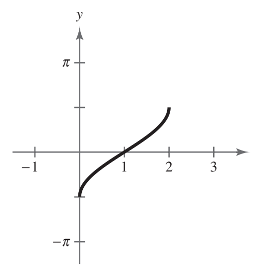


77. $f(x) = \arcsin(x - 1)$

Domain: $0 \le x \le 2$

Range: $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$

This is the graph of $g(x) = \arcsin(x)$ shifted one unit to the right.

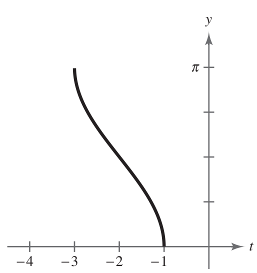


78. $g(t) = \arccos(t + 2)$

Domain: $-3 \le t \le -1$

Range: $0 \le y \le \pi$

This is the graph of $y = \arccos t$ shifted two units to the left.

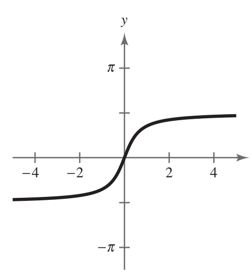


79. $f(x) = \arctan 2x$

Domain: all real numbers

Range: $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

This is the graph of $g(x) = \arctan(x)$ with a horizontal stretch of a factor of 2.

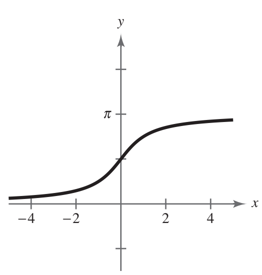


80. $f(x) = \dfrac{\pi}{2} + \arctan x$

Domain: all real numbers

Range: $0 < y \le \pi$

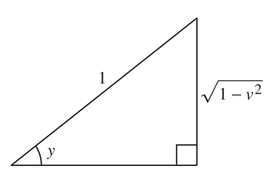
This is the graph of $y = \arctan x$ shifted upward $\pi/2$ units.

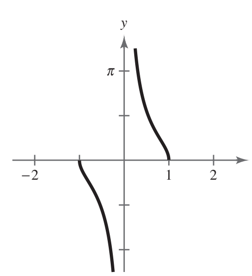


81. $h(v) = \tan(\arccos v) = \dfrac{\sqrt{1 - v^2}}{v}$

Domain: $-1 \le v \le 1, v \ne 0$

Range: all real numbers

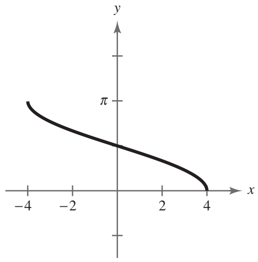




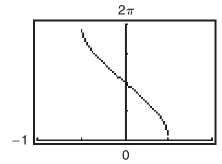
82. $f(x) = \arccos \dfrac{x}{4}$

Domain: $-4 \le x \le 4$

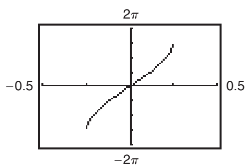
Range: $0 \le y \le \pi$



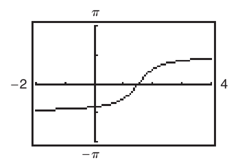
83. $f(x) = 2 \arccos(2x)$



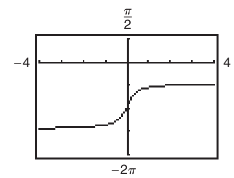
84. $f(x) = \pi \arcsin(4x)$



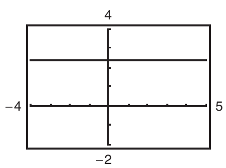
85. $f(x) = \arctan(2x - 3)$



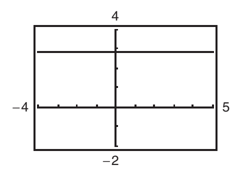
86. $f(x) = -3 + \arctan(\pi x)$



87. $f(x) = \pi - \arcsin\left(\dfrac{2}{3}\right) \approx 2.412$



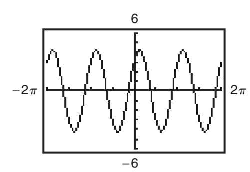
88. $f(x) = \dfrac{\pi}{2} + \arccos\left(\dfrac{1}{\pi}\right) \approx 2.82$



89. $f(t) = 3 \cos 2t + 3 \sin 2t = \sqrt{3^2 + 3^2} \sin\left(2t + \arctan \dfrac{3}{3}\right)$

$$= 3\sqrt{2} \sin(2t + \arctan 1)$$

$$= 3\sqrt{2} \sin\left(2t + \frac{\pi}{4}\right)$$

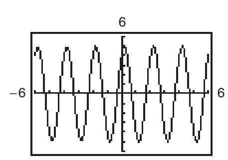


The graph implies that the identity is true.

90. $f(t) = 4 \cos \pi t + 3 \sin \pi t$

$$= \sqrt{4^2 + 3^2} \sin\left(\pi t + \arctan \frac{4}{3}\right)$$

$$= 5 \sin\left(\pi t + \arctan \frac{4}{3}\right)$$



The graph implies that

$A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \sin\left(\omega t + \arctan \dfrac{A}{B}\right)$

is true.

91. (a) $\sin \theta = \dfrac{5}{s}$

$$\theta = \arcsin \frac{5}{s}$$

(b) $s = 40$: $\theta = \arcsin \dfrac{5}{40} \approx 0.13$

$s = 20$: $\theta = \arcsin \dfrac{5}{20} \approx 0.25$

92. (a) $\tan \theta = \dfrac{s}{750}$

$\theta = \arctan \dfrac{s}{750}$

(b) When $s = 300$,

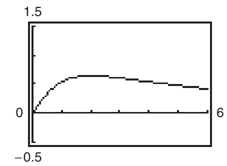
$\theta = \arctan \dfrac{300}{750} \approx 0.38 \approx 21.8°$.

When $s = 1200$,

$\theta = \arctan \dfrac{1200}{750} \approx 1.01 \approx 58.0°$.

93. $\beta = \arctan \dfrac{3x}{x^2 + 4}$

(a)



(b) β is maximum when $x = 2$ feet.

(c) The graph has a horizontal asymptote at $\beta = 0$. As x increases, β decreases.

94. (a) $\tan \theta = \dfrac{11}{17}$

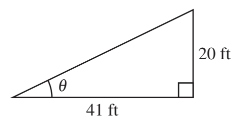
$\theta = \arctan \dfrac{11}{17} \approx 0.5743 \approx 32.9°$

(b) $r = \dfrac{1}{2}(40) = 20$

$\tan \theta = \dfrac{h}{r} = \dfrac{h}{20}$

$h = 20 \tan \theta = 20 \cdot \dfrac{11}{17} \approx 12.94$ feet

95.



(a) $\tan \theta = \dfrac{20}{41}$

$\theta = \arctan\left(\dfrac{20}{41}\right) \approx 26.0°$

(b) $\tan 26° = \dfrac{h}{50}$

$h = 50 \tan 26° \approx 24.39$ feet

96. (a) $\tan \theta = \dfrac{6}{x}$

$\theta = \arctan \dfrac{6}{x}$

(b) $x = 7$ miles

$\theta = \arctan \dfrac{6}{7} \approx 0.71 \approx 40.6°$

$x = 1$ mile

$\theta = \arctan \dfrac{6}{1} \approx 1.41 \approx 80.5°$

97. (a) $\tan \theta = \dfrac{x}{20}$

$\theta = \arctan \dfrac{x}{20}$

(b) $x = 5$: $\theta = \arctan \dfrac{5}{20} \approx 14.0°$

$x = 12$: $\theta = \arctan \dfrac{12}{20} \approx 31.0°$

98. False.

$\dfrac{5\pi}{6}$ is not in the range of $\arcsin(x)$.

$\arcsin \dfrac{1}{2} = \dfrac{\pi}{6}$

99. False.

$\dfrac{5\pi}{4}$ is not in the range of the arctangent function.

$\arctan 1 = \dfrac{\pi}{4}$

100. False.

$\arctan x$ is defined for all real x, but $\arcsin x$ and $\arccos x$ require $-1 \le x \le 1$.

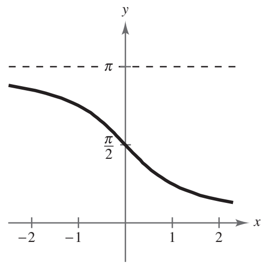
Also, for example, $\arctan 1 \neq \dfrac{\arcsin 1}{\arccos 1}$.

Since $\arctan 1 = \dfrac{\pi}{4}$, but $\dfrac{\arcsin 1}{\arccos 1} = \dfrac{\pi/2}{0} =$ undefined.

101. $y = \operatorname{arccot} x$ if and only if $\cot y = x$.

Domain: $-\infty < x < \infty$

Range: $0 < x < \pi$

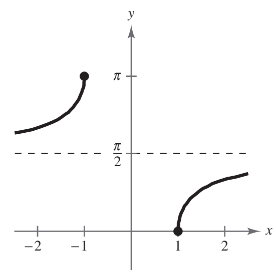


102. $y = \operatorname{arcsec} x$ if and only if $\sec y = x$ where

$x \le -1 \cup x \ge 1$ and $0 \le y < \dfrac{\pi}{2}$ and $\dfrac{\pi}{2} < y \le \pi$.

The domain of $y = \operatorname{arcsec} x$ is $(-\infty, -1] \cup [1, \infty)$

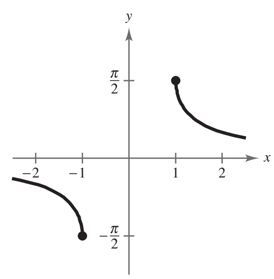
and the range is $\left[0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \pi\right]$.



103. $y = \operatorname{arccsc} x$ if and only if $\csc y = x$.

Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $\left[-\dfrac{\pi}{2}, 0\right) \cup \left(0, \dfrac{\pi}{2}\right]$



104. (a) $y = \operatorname{arcsec} \sqrt{2} \implies \sec y = \sqrt{2}$ and $0 \le y < \dfrac{\pi}{2} \cup \dfrac{\pi}{2} < y \le \pi \implies y = \dfrac{\pi}{4}$

(b) $y = \operatorname{arcsec} 1 \implies \sec y = 1$ and $0 \le y < \dfrac{\pi}{2} \cup \dfrac{\pi}{2} < y \le \pi \implies y = 0$

(c) $y = \operatorname{arccot}(-\sqrt{3}) \implies \cot y = -\sqrt{3}$ and $0 < y < \pi \implies y = \dfrac{5\pi}{6}$

(d) $y = \operatorname{arccsc} 2 \implies \csc y = 2$ and $-\dfrac{\pi}{2} \le y < 0 \cup 0 < y \le \dfrac{\pi}{2} \implies y = \dfrac{\pi}{6}$

105. Area $= \arctan b - \arctan a$

(a) $a = 0, b = 1$

$$\text{Area} = \arctan 1 - \arctan 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

(b) $a = -1, b = 1$

$$\text{Area} = \arctan 1 - \arctan(-1)$$
$$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

(c) $a = 0, b = 3$

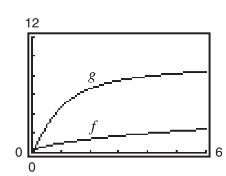
$$\text{Area} = \arctan 3 - \arctan 0$$
$$\approx 1.25 - 0 = 1.25$$

(d) $a = -1, b = 3$

$$\text{Area} = \arctan 3 - \arctan(-1)$$
$$\approx 1.25 - \left(-\frac{\pi}{4}\right) \approx 2.03$$

106. $f(x) = \sqrt{x}$

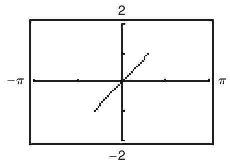
$g(x) = 6 \arctan x$

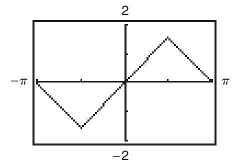


As x increases to infinity, g approaches 3π, but f has no maximum. Using the solve feature of the graphing utility, you find $a \approx 87.54$.

107. $f(x) = \sin(x), f^{-1}(x) = \arcsin(x)$

(a) $f \cdot f^{-1} = \sin(\arcsin x)$ $\qquad$ $f^{-1} \cdot f = \arcsin(\sin x)$





(b) The graphs coincide with the graph of $y = x$ only for certain values of x.

$f \cdot f^{-1} = x$ over its entire domain, $-1 \le x \le 1$.

$f^{-1} \cdot f = x$ over the region $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$, corresponding to the region where $\sin x$ is one-to-one and thus has an inverse.

108. (a) Let $y = \arcsin(-x)$. Then,

$$\begin{aligned} \sin y &= -x \\ -\sin y &= x \\ \sin(-y) &= x \\ -y &= \arcsin x \\ y &= -\arcsin x. \end{aligned}$$

Therefore, $\arcsin(-x) = -\arcsin x$.

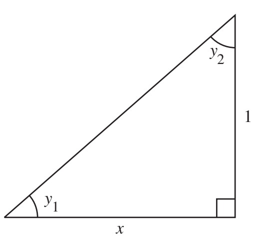
(b) Let $y = \arctan(-x)$. Then,

$$\begin{aligned} \tan y &= -x, -\frac{\pi}{2} < y < \frac{\pi}{2} \\ -\tan y &= x \\ \tan(-y) &= x, -\frac{\pi}{2} < -y < \frac{\pi}{2} \\ \arctan(\tan(-y)) &= \arctan x \\ -y &= \arctan x \\ y &= -\arctan x \end{aligned}$$

Thus, $\arctan(-x) = -\arctan(x)$.

(c) Let $y_2 = \dfrac{\pi}{2} - y_1$.

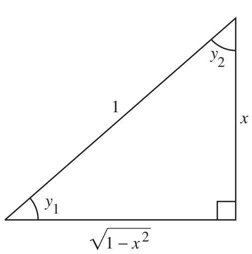
$$\begin{aligned} \arctan x + \arctan \frac{1}{x} &= y_1 + y_2 \\ &= y_1 + \left(\frac{\pi}{2} - y_1\right) = \frac{\pi}{2} \end{aligned}$$



(d) Let $\alpha = \arcsin x$ and $\beta = \arccos x$, then $\sin \alpha = x$ and $\cos \beta = x$. Thus, $\sin \alpha = \cos \beta$ which implies that α and β are complementary angles and we have

$$\begin{aligned} \alpha + \beta &= \frac{\pi}{2} \\ \arcsin x + \arccos x &= \frac{\pi}{2}. \end{aligned}$$

(e) $\arcsin x = \arcsin \dfrac{x}{1} = \arctan \dfrac{x}{\sqrt{1 - x^2}}$



109. $(8.2)^{3.4} \approx 1279.284$

110. $10(14)^{-2} = \dfrac{10}{14^2} = \dfrac{10}{196} \approx 0.051$

111. $(1.1)^{50} \approx 117.391$

112. $16^{-2\pi} = \dfrac{1}{16^{2\pi}} \approx 2.718 \times 10^{-8}$

113.

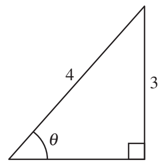
$$\sin\theta = \frac{3}{4} = \frac{\text{opp}}{\text{hyp}}$$

$$(\text{adj})^2 + (3)^2 = (4)^2$$

$$(\text{adj})^2 + 9 = 16$$

$$(\text{adj})^2 = 7$$

$$\text{adj} = \sqrt{7}$$



$$\cos\theta = \frac{\sqrt{7}}{4}$$

$$\tan\theta = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

$$\cot\theta = \frac{\sqrt{7}}{3}$$

$$\sec\theta = \frac{4}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$$

$$\csc\theta = \frac{4}{3}$$

114. $\tan\theta = 2$

$$\text{hyp} = \sqrt{1^2 + 2^2} = \sqrt{5}$$

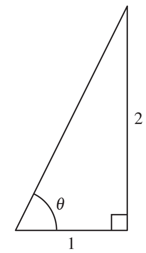
$$\cos\theta = \frac{1}{\sqrt{5}}$$

$$\sin\theta = \frac{2}{\sqrt{5}}$$

$$\cot\theta = \frac{1}{2}$$

$$\sec\theta = \sqrt{5}$$

$$\csc\theta = \frac{1}{2}\sqrt{5}$$



115.

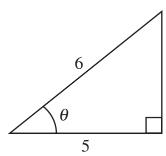
$$\cos\theta = \frac{5}{6} = \frac{\text{adj}}{\text{hyp}}$$

$$(\text{opp})^2 + (5)^2 = (6)^2$$

$$(\text{opp})^2 + 25 = 36$$

$$(\text{opp})^2 = 11$$

$$\text{opp} = \sqrt{11}$$



$$\sin\theta = \frac{\sqrt{11}}{6}$$

$$\tan\theta = \frac{\sqrt{11}}{5}$$

$$\cot\theta = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$$

$$\sec\theta = \frac{6}{5}$$

$$\csc\theta = \frac{6}{\sqrt{11}} = \frac{6\sqrt{11}}{11}$$

116. $\sec\theta = 3$

$$\text{opp} = \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$$

$$\cos\theta = \frac{1}{3}$$

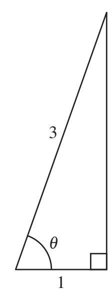
$$\sin\theta = \frac{2\sqrt{2}}{3}$$

$$\tan\theta = 2\sqrt{2}$$

$$\sec\theta = 3$$

$$\csc\theta = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\cot\theta = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$



117. Let $x =$ the number of people presently in the group. Each person's share is now $250{,}000/x$. If two more join the group, each person's share would then be $250{,}000/(x + 2)$.

$$\text{Share per person with two more people} = \text{Original share per person} - 6250$$

$$\frac{250{,}000}{x+2} = \frac{250{,}000}{x} - 6250$$

$$250{,}000x = 250{,}000(x + 2) - 6250x(x + 2)$$

$$250{,}000x = 250{,}000x + 500{,}000 - 6250x^2 - 12500x$$

$$6250x^2 + 12500x - 500{,}000 = 0$$

$$6250(x^2 + 2x - 80) = 0$$

$$6250(x + 10)(x - 8) = 0$$

$$x = -10 \quad \text{or} \quad x = 8$$

$x = -10$ is not possible.

There were 8 people in the original group.

118. Rate downstream: $18 + x$

Rate upstream: $18 - x$

rate $\times$ time $=$ distance $\implies t = \dfrac{d}{r}$

(Time to go upstream) $+$ (Time to go downstream) $= 4$

$$\frac{35}{18-x} + \frac{35}{18+x} = 4$$

$$35(18 + x) + 35(18 - x) = 4(18 - x)(18 + x)$$

$$630 + 35x + 630 - 35x = 4(324 - x^2)$$

$$1260 = 4(324 - x^2)$$

$$315 = 324 - x^2$$

$$x^2 = 9$$

$$x = \pm 3$$

The speed of the current is 3 miles per hour.

119. (a) $A = 15{,}000\left(1 + \dfrac{0.035}{4}\right)^{(4)(10)} \approx \$21{,}253.63$

(b) $A = 15{,}000\left(1 + \dfrac{0.035}{12}\right)^{(12)(10)} \approx \$21{,}275.17$

(c) $A = 15{,}000\left(1 + \dfrac{0.035}{365}\right)^{(365)(10)} \approx \$21{,}285.66$

(d) $A = 15{,}000e^{(0.035)(10)} \approx \$21{,}286.01$

120. Data: $(2, 742{,}000)$, $(4, 632{,}000)$

To find: $(8, y)$

Assume: $P = P_0 \cdot e^{-rt}$

$$742{,}000 = P_0 e^{-r\cdot 2}$$

$$632{,}000 = P_0 e^{-r\cdot 4}$$

Then: $e^{-r\cdot 2} = \dfrac{P_0 e^{-r\cdot 4}}{P_0 e^{-r\cdot 2}} = \dfrac{632}{742}$

$$\begin{aligned} y &= P_0 e^{-r\cdot 8} = P_0 e^{-r\cdot 4} \cdot e^{-r\cdot 4} \\ &= 632{,}000 \cdot (e^{-r\cdot 2})^2 \\ &= 632{,}000 \cdot \left(\frac{632}{742}\right)^2 \\ &= 458{,}504.31 \end{aligned}$$

Section 4.8 Applications and Models

- You should be able to solve right triangles.
- You should be able to solve right triangle applications.
- You should be able to solve applications of simple harmonic motion.

Vocabulary Check

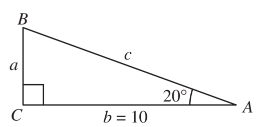
1. elevation; depression
2. bearing
3. harmonic motion

1. Given: $A = 20°,\ b = 10$

$$\tan A = \frac{a}{b} \Rightarrow a = b\tan A = 10\tan 20° \approx 3.64$$

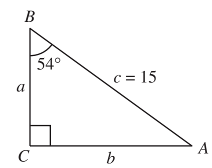
$$\cos A = \frac{b}{c} \Rightarrow c = \frac{b}{\cos A} = \frac{10}{\cos 20°} \approx 10.64$$

$$B = 90° - 20° = 70°$$



2. Given: $B = 54°, c = 15$

$$A = 90° - B$$
$$= 90° - 54° = 36°$$



$$\sin B = \frac{b}{c} \Rightarrow b = c\sin B$$
$$= 15\sin 54° \approx 12.14$$

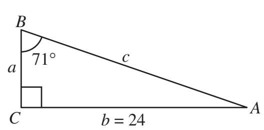
$$\cos B = \frac{a}{c} \Rightarrow a = c\cos B = 15\cos 54° \approx 8.82$$

3. Given: $B = 71°,\ b = 24$

$$\tan B = \frac{b}{a} \Rightarrow a = \frac{b}{\tan B} = \frac{24}{\tan 71°} \approx 8.26$$

$$\sin B = \frac{b}{c} \Rightarrow c = \frac{b}{\sin B} = \frac{24}{\sin 71°} \approx 25.38$$

$$A = 90° - 71° = 19°$$

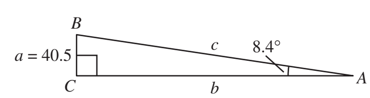


4. Given: $A = 8.4°, a = 40.5$

$$B = 90° - A$$
$$= 90° - 8.4° = 81.6°$$

$$\tan A = \frac{a}{b} \Rightarrow b = \frac{a}{\tan A}$$
$$= \frac{40.5}{\tan 8.4°} \approx 274.27$$

$$\sin A = \frac{a}{c} \Rightarrow c = \frac{a}{\sin A} = \frac{40.5}{\sin 8.4°} \approx 277.24$$

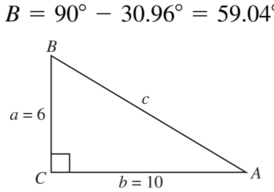


5. Given: $a = 6,\ b = 10$

$$c^2 = a^2 + b^2 \Rightarrow c = \sqrt{36 + 100}$$
$$= 2\sqrt{34} \approx 11.66$$

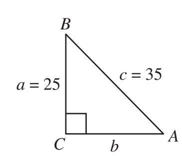
$$\tan A = \frac{a}{b} = \frac{6}{10} \Rightarrow A = \arctan\frac{3}{5} \approx 30.96°$$

$$B = 90° - 30.96° = 59.04°$$



6. Given: $a = 25, c = 35$

$$b = \sqrt{c^2 - a^2}$$
$$= \sqrt{35^2 - 25^2}$$
$$= \sqrt{600} \approx 24.49$$

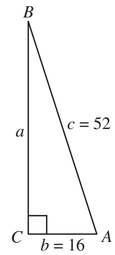


$$\sin A = \frac{a}{c} \Rightarrow A = \arcsin\frac{a}{c}$$
$$= \arcsin\frac{25}{35} \approx 45.58°$$

$$\cos B = \frac{a}{c} \Rightarrow B = \arccos\frac{a}{c} = \arccos\frac{25}{35} \approx 44.42°$$

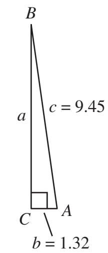
7. Given: $b = 16,\ c = 52$

$$a = \sqrt{52^2 - 16^2}$$
$$= \sqrt{2448} = 12\sqrt{17} \approx 49.48$$
$$\cos A = \frac{16}{52}$$
$$A = \arccos \frac{16}{52} \approx 72.08°$$
$$B = 90° - 72.08° \approx 17.92°$$



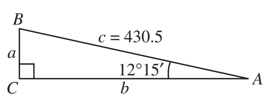
8. Given: $b = 1.32, c = 9.45$

$$a = \sqrt{c^2 - b^2} = \sqrt{87.5601} \approx 9.36$$
$$\cos A = \frac{b}{c} \implies A = \arccos \frac{b}{c} = \arccos \frac{1.32}{9.45} \approx 81.97°$$
$$\sin B = \frac{b}{c} \implies B = \arcsin \frac{b}{c}$$
$$= \arcsin \frac{1.32}{9.45}$$
$$\approx 8.03°$$



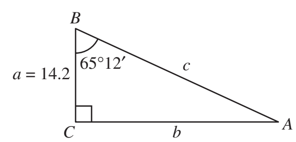
9. Given: $A = 12° \ 15',\ c = 430.5$

$$B = 90° - 12° \ 15' = 77° \ 45'$$
$$\sin 12° 15' = \frac{a}{430.5}$$
$$a = 430.5 \sin 12° 15' \approx 91.34$$
$$\cos 12° 15' = \frac{b}{430.5}$$
$$b = 430.5 \cos 12° 15' \approx 420.70$$



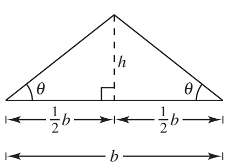
10. Given: $B = 65° \ 12', a = 14.2$

$$A = 90° - B = 90° - 65° \ 12' = 24° \ 48'$$
$$\cos B = \frac{a}{c} \implies c = \frac{a}{\cos B} = \frac{14.2}{\cos 65° \ 12'} \approx 33.85$$
$$\tan B = \frac{b}{a} \implies b = a \tan B = 14.2 \tan 65° \ 12' \approx 30.73$$



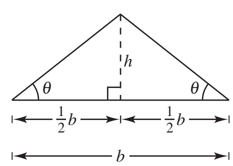
11. $\tan \theta = \dfrac{h}{(1/2)b} \implies h = \dfrac{1}{2} b \tan \theta$

$$h = \frac{1}{2}(4) \tan 52° \approx 2.56 \text{ inches}$$



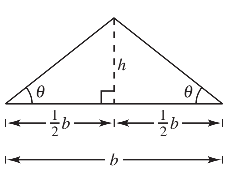
12. $\tan \theta = \dfrac{h}{(1/2)b} \implies h = \dfrac{1}{2} b \tan \theta$

$$h = \frac{1}{2}(10) \tan 18° \approx 1.62 \text{ meters}$$



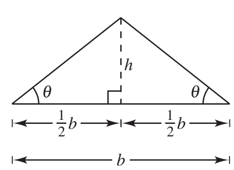
13. $\tan \theta = \dfrac{h}{(1/2)b} \implies h = \dfrac{1}{2} b \tan \theta$

$$h = \frac{1}{2}(46) \tan 41° \approx 19.99 \text{ inches}$$



14. $\tan \theta = \dfrac{h}{(1/2)b} \implies h = \dfrac{1}{2} b \tan \theta$

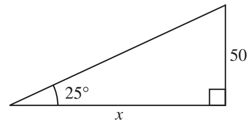
$$h = \frac{1}{2}(11) \tan 27° \approx 2.80 \text{ feet}$$



15. $\tan 25° = \dfrac{50}{x}$

$x = \dfrac{50}{\tan 25°}$

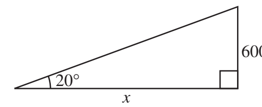
≈ 107.2 feet



16. $\tan 20° = \dfrac{600}{x}$

$x = \dfrac{600}{\tan 20}$

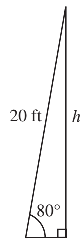
≈ 1648.5 feet



17. $\sin 80° = \dfrac{h}{20}$

$20 \sin 80° = h$

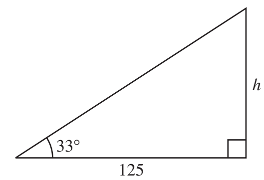
$h \approx 19.7$ feet



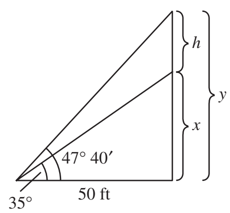
18. $\tan 33° = \dfrac{h}{125}$

$h = 125 \tan 33$

≈ 81.2 feet



19. (a)



(b) Let the height of the church $= x$ and the height of the church and steeple $= y$. Then,

$\tan 35° = \dfrac{x}{50}$ and $\tan 47°40' = \dfrac{y}{50}$

$x = 50 \tan 35°$ and $y = 50 \tan 47°40'$

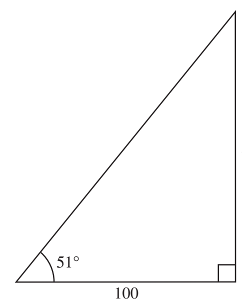
$h = y - x = 50(\tan 47°40' - \tan 35°)$.

(c) $h \approx 19.9$ feet

20. $\tan 51° = \dfrac{h}{100}$

$h = 100 \tan 51°$

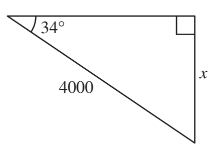
≈ 123.5 feet



21. $\sin 34° = \dfrac{x}{4000}$

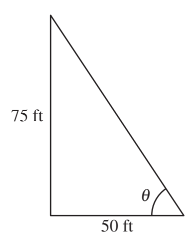
$x = 4000 \sin 34°$

≈ 2236.8 feet

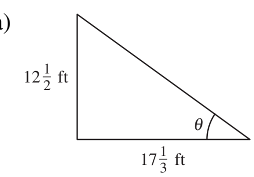


22. $\tan \theta = \dfrac{75}{50}$

$\theta = \arctan \dfrac{3}{2} \approx 56.3°$



23. (a)



(b) $\tan \theta = \dfrac{12\frac{1}{2}}{17\frac{1}{3}}$

(c) $\theta = \arctan \dfrac{12\frac{1}{2}}{17\frac{1}{3}} \approx 35.8°$

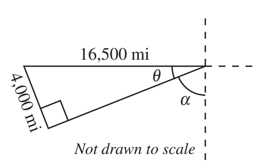
The angle of elevation of the sum is 35.8°.

24. $12{,}500 + 4000 = 16{,}500$

$\sin \theta = \dfrac{4000}{16{,}500}$

$\theta = \arcsin\left(\dfrac{4000}{16{,}500}\right)$

$\theta \approx 14.03°$



Angle of depression $= \alpha \approx 90° - 14.03° = 75.97°$

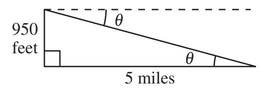
25. $1200 \text{ feet} + 150 \text{ feet} - 400 \text{ feet} = 950 \text{ feet}$

$$5 \text{ miles} = 5 \text{ miles}\left(\frac{5280 \text{ feet}}{1 \text{ mile}}\right) = 26{,}400 \text{ feet}$$

$$\tan\theta = \frac{950}{26{,}400}$$

$$\theta = \arctan\left(\frac{950}{26{,}400}\right) \approx 2.06°$$

Not drawn to scale



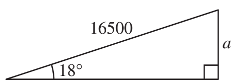
26. (a) Since the airplane speed is

$$\left(275\frac{\text{ft}}{\text{sec}}\right)\left(60\frac{\text{sec}}{\text{min}}\right) = 16{,}500\frac{\text{ft}}{\text{min}},$$

after one minute its distance travelled is 16,500 feet.

$$\sin 18° = \frac{a}{16{,}500}$$

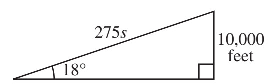
$$a = 16{,}500 \sin 18° \approx 5099 \text{ ft}$$



(b) $\sin 18° = \dfrac{10{,}000}{275s}$

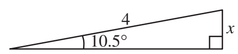
$$s = \frac{10{,}000}{275(\sin 18°)}$$

$$\approx 117.7 \text{ seconds}$$

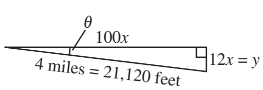


27. $\sin 10.5° = \dfrac{x}{4}$

$$x = 4 \sin 10.5° \approx 0.73 \text{ mile}$$



28.



Angle of grade: $\tan\theta = \dfrac{12x}{100x}$

$$\theta = \arctan 0.12 \approx 6.8°$$

Change in elevation:

$$\sin\theta = \frac{y}{21{,}120}$$

$$y = 21{,}120 \sin\theta$$

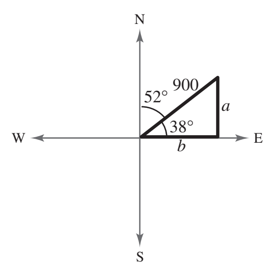
$$= 21{,}120 \sin(\arctan 0.12)$$

$$\approx 2516.3 \text{ feet}$$

29. The plane has traveled $1.5(600) = 900$ miles.

$$\sin 38° = \frac{a}{900} \implies a \approx 554 \text{ miles north}$$

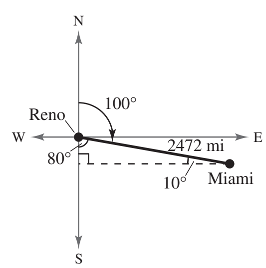
$$\cos 38° = \frac{b}{900} \implies b \approx 709 \text{ miles east}$$

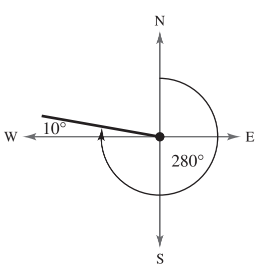


30. (a) Reno is $2472 \sin 10° = 429$ miles N of Miami.

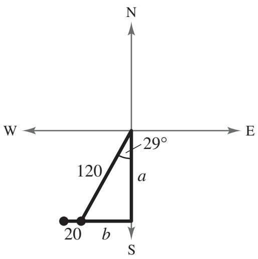
Reno is $2472 \cos 10° = 2434$ miles W of Miami.

(b) The return heading is 280°.





31.



(a) $\cos 29° = \dfrac{a}{120} \implies a \approx 104.95$ nautical miles south

$\sin 29° = \dfrac{b}{120} \implies b \approx 58.18$ nautical miles west

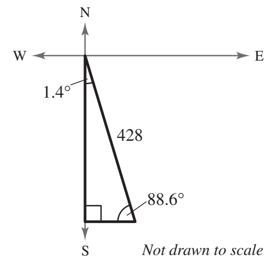
(b) $\tan\theta = \dfrac{20 + b}{a} \approx \dfrac{78.18}{104.95} \implies \theta \approx 36.7°$

Bearing: S 36.7° W

Distance: $d \approx \sqrt{104.95^2 + 78.18^2}$

≈ 130.9 nautical miles from port

32.



(a) $t = \dfrac{428}{20} = 21.4$ hours

(b) After 12 hours, the yacht will have traveled 240 nautical miles.

$240 \sin 1.4° \approx 5.9$ miles E

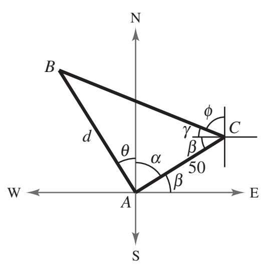
$240 \cos 1.4° \approx 239.9$ miles S

(c) Bearing from N is 178.6°.

33. $\theta = 32°,\ \phi = 68°$

(a) $\alpha = 90° - 32° = 58°$

Bearing from A to C: N 58º E



(b) $\beta = \theta = 32°$

$\gamma = 90° - \phi = 22°$

$C = \beta + \gamma = 54°$

$\tan C = \dfrac{d}{50} \implies \tan 54°$

$= \dfrac{d}{50} \implies d \approx 68.82$ meters

34. $\tan 14° = \dfrac{d}{x} \implies x = d \cot 14°$

$$\tan 34° = \frac{d}{y} = \frac{d}{30 - x}$$

$$= \frac{d}{30 - d\cot 14°}$$

$$\cot 34° = \frac{30 - d\cot 14°}{d}$$

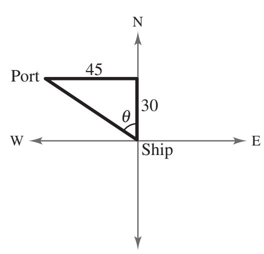
$$d \cot 34° = 30 - d\cot 14°$$

$$d = \frac{30}{\cot 34° + \cot 14°}$$

≈ 5.46 kilometers

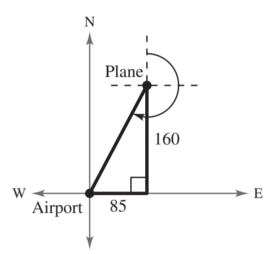
35. $\tan\theta = \dfrac{45}{30} \implies \theta \approx 56.3°$

Bearing: N 56.3° W



36. Bearing $= 180° + \arctan\left(\dfrac{85}{160}\right)$

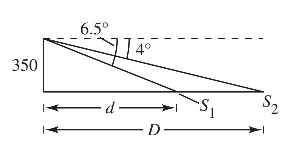
$= 208.0°$ or $528°$ W



37. $\tan 6.5° = \dfrac{350}{d} \implies d \approx 3071.91$ ft

$\tan 4° = \dfrac{350}{D} \implies D \approx 5005.23$ ft

Distance between ships: $D - d \approx 1933.3$ ft

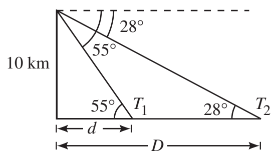


38. $\cot 55 = \dfrac{d}{10} \implies d \approx 7$ kilometers

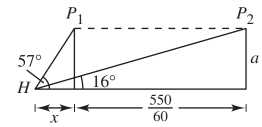
$\cot 28^\circ = \dfrac{D}{10} \implies D \approx 18.8$ kilometers

Distance between towns:

$D - d = 18.8 - 7 = 11.8$ kilometers



39. $\tan 57^\circ = \dfrac{a}{x} \implies x = a \cot 57^\circ$



$$\tan 16^\circ = \frac{a}{x + (55/6)}$$

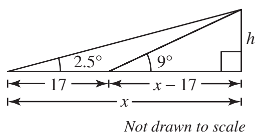
$$\tan 16^\circ = \frac{a}{a \cot 57^\circ + (55/6)}$$

$$\cot 16^\circ = \frac{a \cot 57^\circ + (55/6)}{a}$$

$$a \cot 16^\circ - a \cot 57^\circ = \frac{55}{6} \implies a \approx 3.23 \text{ miles}$$

$$\approx 17{,}054 \text{ ft}$$

40. $\tan 2.5^\circ = \dfrac{h}{x}$



$$x = \frac{h}{\tan 2.5^\circ}$$

$$\tan 9^\circ = \frac{h}{x - 17}$$

$$x = \frac{h}{\tan 9^\circ} + 17$$

$$\frac{h}{\tan 2.5^\circ} = \frac{h}{\tan 9^\circ} + 17$$

$$h = \frac{17}{\left(\dfrac{1}{\tan 2.5^\circ} - \dfrac{1}{\tan 9^\circ}\right)} \approx 1.025 \text{ miles}$$

$$\approx 5410 \text{ feet}$$

41. L_1: $3x - 2y = 5 \implies y = \dfrac{3}{2}x - \dfrac{5}{2} \implies m_1 = \dfrac{3}{2}$

L_2: $x + y = 1 \implies y = -x + 1 \implies m_2 = -1$

$$\tan \alpha = \left|\frac{-1 - (3/2)}{1 + (-1)(3/2)}\right| = \left|\frac{-5/2}{-1/2}\right| = 5$$

$$\alpha = \arctan 5 \approx 78.7^\circ$$

42. $L_1 = 2x - y = 8 \implies m_1 = 2$

$L_2 = x - 5y = -4 \implies m_2 = \dfrac{1}{5}$

$$\tan \alpha = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right|$$

$$\alpha = \arctan\left|\frac{m_2 - m_1}{1 + m_2 m_1}\right| = \arctan\left|\frac{(1/5) - 2}{1 + (1/5)(2)}\right|$$

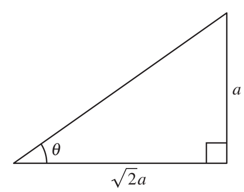
$$= \arctan\left(\frac{9}{7}\right) \approx 52.1^\circ$$

43. The diagonal of the base has a length of $\sqrt{a^2 + a^2} = \sqrt{2}a$. Now, we have

$$\tan \theta = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

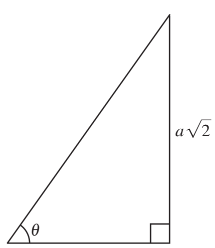
$$\theta = \arctan \frac{1}{\sqrt{2}}$$

$$\theta \approx 35.3^\circ.$$



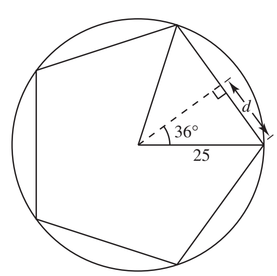
44. $\tan \theta = \dfrac{a\sqrt{2}}{a} = \sqrt{2}$

$$\theta = \arctan \sqrt{2} \approx 54.7^\circ$$

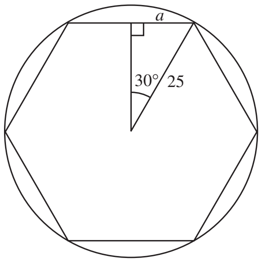


45. $\sin 36^\circ = \dfrac{d}{25} \implies d \approx 14.69$

Length of side: $2d \approx 29.4$ inches



46.

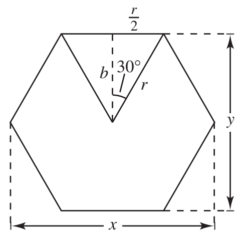


$$\sin 30° = \frac{a}{25}$$

$$a = 25 \sin 30° = 12.5$$

$$\text{Length of side} = 2a = 2(12.5)$$
$$= 25 \text{ inches}$$

47.



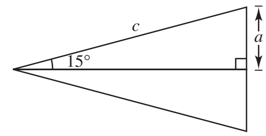
$$\cos 30° = \frac{b}{r}$$

$$b = r \cos 30°$$

$$b = \frac{\sqrt{3}r}{2}$$

$$y = 2b = 2\left(\frac{\sqrt{3}r}{2}\right) = \sqrt{3}r$$

48.



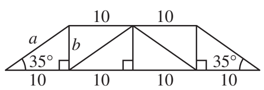
$$c = \frac{35}{2} = 17.5$$

$$\sin 15° = \frac{a}{c}$$

$$a = c \sin 15° = 17.5 \sin 15°$$
$$\approx 4.53$$

$$\text{Distance} = 2a \approx 9.06 \text{ centimeters}$$

49.



$$\tan 35° = \frac{b}{10}$$

$$b = 10 \tan 35° \approx 7$$

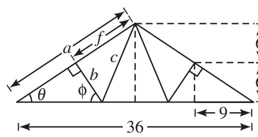
$$\cos 35° = \frac{10}{a}$$

$$a = \frac{10}{\cos 35°} \approx 12.2$$

50. $\tan \theta = \dfrac{12}{18}$

$$\theta = \arctan \frac{2}{3} = 0.588 \text{ rad} \approx 33.7°$$

$$\cos \theta = \frac{18}{a}$$



$$a = \frac{18}{\cos \theta} \approx 21.6 \text{ feet}$$

$$f \approx \frac{21.6}{2} = 10.8 \text{ feet}$$

$$\phi \approx 90 - 33.7 = 56.3°$$

$$\sin \phi = \frac{6}{b}$$

$$b = \frac{6}{\sin \phi} \approx 7.2 \text{ feet}$$

$$c = \sqrt{10.8^2 + 7.2^2} \approx 13 \text{ feet}$$

51. $d = 0$ when $t = 0$, $a = 4$, period $= 2$

Use $d = a \sin \omega t$ since $d = 0$ when $t = 0$.

$$\frac{2\pi}{\omega} = 2 \implies \omega = \pi$$

Thus, $d = 4 \sin(\pi t)$.

52. Displacement at $t = 0$ is $0 \implies d = a \sin \omega t$.

Amplitude: $|a| = 3$

Period: $\dfrac{2\pi}{\omega} = 6 \implies \omega = \dfrac{\pi}{3}$

$$d = 3 \sin\left(\frac{\pi t}{3}\right)$$

53. $d = 3$ when $t = 0$, $a = 3$, period $= 1.5$

Use $d = a \cos \omega t$ since $d = 3$ when $t = 0$.

$$\frac{2\pi}{\omega} = 1.5 \implies \omega = \frac{4\pi}{3}$$

Thus, $d = 3 \cos\left(\dfrac{4\pi}{3}t\right) = 3 \cos\left(\dfrac{4\pi t}{3}\right)$.

54. Displacement at $t = 0$ is $2 \implies d = a \cos \omega t$.

Amplitude: $|a| = 2$

Period: $\dfrac{2\pi}{\omega} = 10 \implies \omega = \dfrac{\pi}{5}$

$$d = 2 \cos\left(\frac{\pi t}{5}\right)$$

55. $d = 4 \cos 8\pi t$

(a) Maximum displacement = amplitude = 4

(b) Frequency $= \dfrac{\omega}{2\pi} = \dfrac{8\pi}{2\pi}$

$= 4$ cycles per unit of time

(c) $d = 4 \cos 40\pi = 4$

(d) $8\pi t = \dfrac{\pi}{2} \implies t = \dfrac{1}{16}$

56. $d = \dfrac{1}{2} \cos 20\pi t$

(a) Maximum displacement: $|a| = \left|\dfrac{1}{2}\right| = \dfrac{1}{2}$

(b) Frequency: $\dfrac{\omega}{2\pi} = \dfrac{20\pi}{2\pi} = 10$ cycles per unit of time

(c) $t = 5 \implies d = \dfrac{1}{2} \cos 100\pi \approx \dfrac{1}{2}$

(d) Least positive value for t for which $d = 0$

$$\frac{1}{2} \cos 20\pi t = 0$$

$$\cos 20\pi t = 0$$

$$20\pi t = \arccos 0$$

$$20\pi t = \frac{\pi}{2}$$

$$t = \frac{\pi}{2} \cdot \frac{1}{20\pi} = \frac{1}{40}$$

57. $d = \dfrac{1}{16} \sin 120\pi t$

(a) Maximum displacement = amplitude $= \dfrac{1}{16}$

(b) Frequency $= \dfrac{\omega}{2\pi} = \dfrac{120\pi}{2\pi}$

$= 60$ cycles per unit of time

(c) $d = \dfrac{1}{16} \sin 600\pi = 0$

(d) $120\pi t = \pi \implies t = \dfrac{1}{120}$

58. $d = \dfrac{1}{64} \sin 792\pi t$

(a) Maximum displacement: $|a| = \left|\dfrac{1}{64}\right| = \dfrac{1}{64}$

(b) Frequency: $\dfrac{\omega}{2\pi} = \dfrac{792\pi}{2\pi} = 396$ cycles per unit of time

(c) $t = 5 \implies d = \dfrac{1}{64} \sin(3960\pi) = 0$

(d) Least positive value for t for which $d = 0$

$$\frac{1}{64} \sin 792\pi t = 0$$

$$\sin 792\pi t = 0$$

$$792\pi t = \arcsin 0$$

$$792\pi t = \pi$$

$$t = \frac{\pi}{792\pi} = \frac{1}{792}$$

59. $d = a \sin \omega t$

Frequency $= \dfrac{\omega}{2\pi}$

$$264 = \frac{\omega}{2\pi}$$

$$\omega = 2\pi(264) = 528\pi$$

60. At $t = 0$, buoy is at its high point $\implies d = a \cos \omega t$.

Distance from high to low $= 2|a| = 3.5$

$$|a| = \frac{7}{4}$$

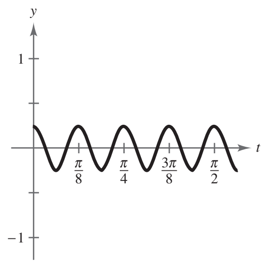
Returns to high point every 10 seconds:

Period: $\dfrac{2\pi}{\omega} = 10 \implies \omega = \dfrac{\pi}{5}$

$$d = \frac{7}{4} \cos \frac{\pi t}{5}$$

61. $y = \frac{1}{4}\cos 16t,\ t > 0$

(a)



(b) Period: $\frac{2\pi}{16} = \frac{\pi}{8}$

(c) $\frac{1}{4}\cos 16t = 0$ when $16t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{32}$

62. (a)

θ	L_1	L_2	$L_1 + L_2$
0.1	$\frac{2}{\sin 0.1}$	$\frac{3}{\cos 0.1}$	23.0
0.2	$\frac{2}{\sin 0.2}$	$\frac{3}{\cos 0.2}$	13.1
0.3	$\frac{2}{\sin 0.3}$	$\frac{3}{\cos 0.3}$	9.9
0.4	$\frac{2}{\sin 0.4}$	$\frac{3}{\cos 0.4}$	8.4

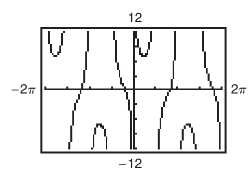
(b)

θ	L_1	L_2	$L_1 + L_2$
0.5	$\frac{2}{\sin 0.5}$	$\frac{3}{\cos 0.5}$	7.6
0.6	$\frac{2}{\sin 0.6}$	$\frac{3}{\cos 0.6}$	7.2
0.7	$\frac{2}{\sin 0.7}$	$\frac{3}{\cos 0.7}$	7.0
0.8	$\frac{2}{\sin 0.8}$	$\frac{3}{\cos 0.8}$	7.1

The minimum length of the elevator is 7.0 meters.

(c) $L = L_1 + L_2 = \frac{2}{\sin\theta} + \frac{3}{\cos\theta}$

(d)



From the graph, it appears that the minimum length is 7.0 meters, which agrees with the estimate of part (b).

63. (a) and (b)

Base 1	Base 2	Altitude	Area
8	$8 + 16\cos 10°$	$8\sin 10°$	22.1
8	$8 + 16\cos 20°$	$8\sin 20°$	42.5
8	$8 + 16\cos 30°$	$8\sin 30°$	59.7
8	$8 + 16\cos 40°$	$8\sin 40°$	72.7
8	$8 + 16\cos 50°$	$8\sin 50°$	80.5
8	$8 + 16\cos 60°$	$8\sin 60°$	83.1
8	$8 + 16\cos 70°$	$8\sin 70°$	80.7

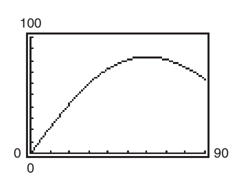
The maximum occurs when $\theta = 60°$ and is approximately 83.1 square feet.

(c) $A(\theta) = [8 + (8 + 16\cos\theta)]\left[\frac{8\sin\theta}{2}\right]$

$= (16 + 16\cos\theta)(4\sin\theta)$

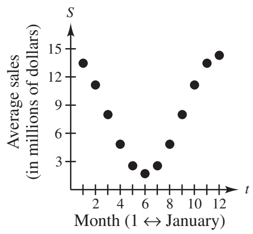
$= 64(1 + \cos\theta)(\sin\theta)$

(d)



The maximum of 83.1 square feet occurs when $\theta = \frac{\pi}{3} = 60°$.

64. (a)



(c) Period: $\dfrac{2\pi}{\pi/6} = 12$

This corresponds to the 12 months in a year. Since the sales of outerwear is seasonal, this is reasonable.

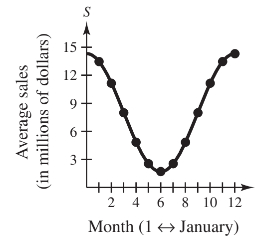
(b) $a = \dfrac{1}{2}(14.3 - 1.7) = 6.3$

$\dfrac{2\pi}{b} = 12 \implies b = \dfrac{\pi}{6}$

Shift: $d = 14.3 - 6.3 = 8$

$S = d + a\cos bt$

$S = 8 + 6.3\cos\left(\dfrac{\pi t}{6}\right)$



Note: Another model is $S = 8 + 6.3\sin\left(\dfrac{\pi t}{6} + \dfrac{\pi}{2}\right)$.

The model is a good fit.

(d) The amplitude represents the maximum displacement from average sales of 8 million dollars. Sales are greatest in December (cold weather + Christmas) and least in June.

65. False. Since the tower is not exactly vertical, a right triangle with sides 191 feet and d is not formed.

66. False. One period is the time for one complete cycle of the motion.

67. No. N 24° E means 24° east of north.

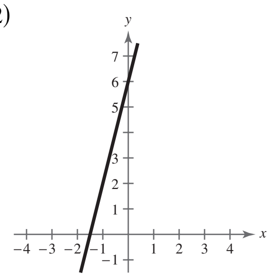
68. Aeronautical bearings are always taken clockwise from North (rather than the acute angle from a north-south line).

69. $m = 4$, passes through $(-1, 2)$

$y - 2 = 4(x - (-1))$

$y - 2 = 4x + 4$

$y = 4x + 6$



70. Linear equation $m = -\frac{1}{2}$ through $\left(\frac{1}{3}, 0\right)$

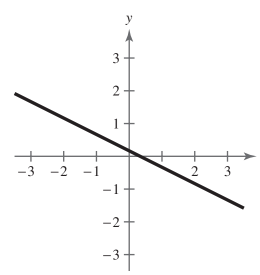
$y = -\frac{1}{2}x + b$

$0 = -\frac{1}{2}\left(\frac{1}{3}\right) + b$

$0 = -\frac{1}{6} + b$

$b = \frac{1}{6}$

$y = -\frac{1}{2}x + \frac{1}{6}$



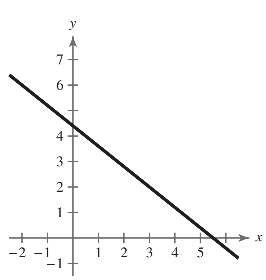
71. Passes through $(-2, 6)$ and $(3, 2)$

$m = \dfrac{2 - 6}{3 - (-2)} = -\dfrac{4}{5}$

$y - 6 = -\dfrac{4}{5}[x - (-2)]$

$y - 6 = -\dfrac{4}{5}x - \dfrac{8}{5}$

$y = -\dfrac{4}{5}x + \dfrac{22}{5}$



72. Linear equation through $\left(\dfrac{1}{4}, -\dfrac{2}{3}\right)$ and $\left(-\dfrac{1}{2}, \dfrac{1}{3}\right)$

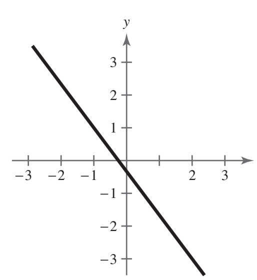
$m = \dfrac{(1/3) - (-2/3)}{(-1/2) - (1/4)}$

$= \dfrac{1}{-3/4}$

$= -\dfrac{4}{3}$

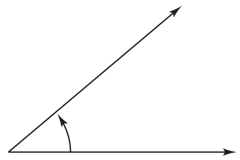
$y + \dfrac{2}{3} = -\dfrac{4}{3}\left(x - \dfrac{1}{4}\right)$

$y = -\dfrac{4}{3}x - \dfrac{1}{3}$

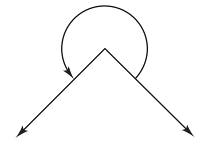


Review Exercises for Chapter 4

1. $\theta \approx 0.5$ radian

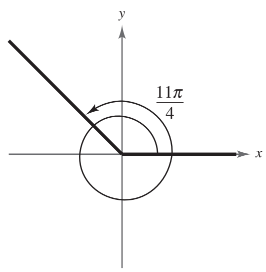


2. $\theta \approx 4.5$ radians



3. $\theta = \dfrac{11\pi}{4}$

(a)



(b) The angle lies in Quadrant II.

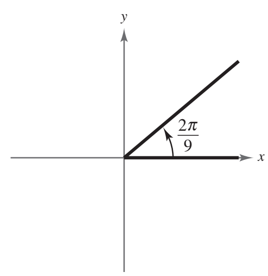
(c) Coterminal angles:

$$\frac{11\pi}{4} - 2\pi = \frac{3\pi}{4}$$

$$\frac{3\pi}{4} - 2\pi = -\frac{5\pi}{4}$$

4. $\theta = \dfrac{2\pi}{9}$

(a)



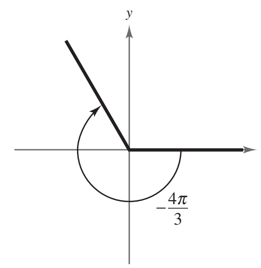
(b) Quadrant I

(c) $\dfrac{2\pi}{9} + 2\pi = \dfrac{20\pi}{9}$

$$\frac{2\pi}{9} - 2\pi = -\frac{16\pi}{9}$$

5. $\theta = -\dfrac{4\pi}{3}$

(a)



(b) The angle lies in Quadrant II.

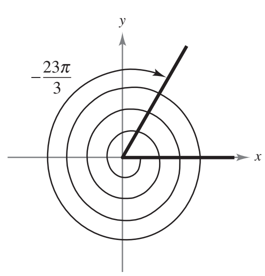
(c) Coterminal angles:

$$-\frac{4\pi}{3} + 2\pi = \frac{2\pi}{3}$$

$$-\frac{4\pi}{3} - 2\pi = -\frac{10\pi}{3}$$

6. $\theta = -\dfrac{23\pi}{3}$

(a)



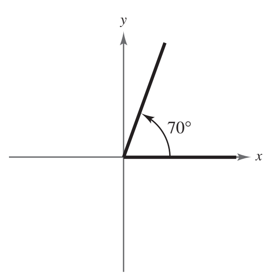
(b) Quadrant I

(c) $-\dfrac{23\pi}{3} + 8\pi = \dfrac{\pi}{3}$

$$-\frac{23\pi}{3} + 2\pi = -\frac{17\pi}{3}$$

7. $\theta = 70°$

(a)



(b) The angle lies in Quadrant I.

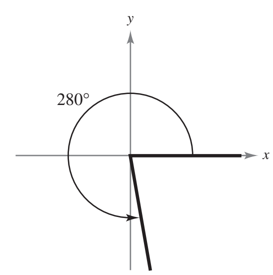
(c) Coterminal angles:

$70° + 360° = 430°$

$70° - 360° = -290°$

8. $\theta = 280°$

(a)



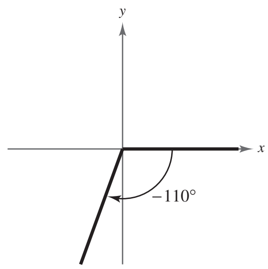
(b) Quadrant IV

(c) $280° + 360° = 640°$

$280° - 360° = -80°$

9. $\theta = -110°$

(a)



(b) The angle lies in Quadrant III.

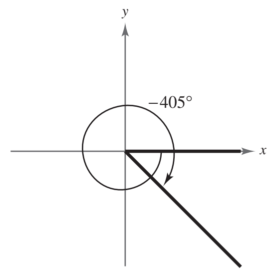
(c) Coterminal angles:

$-110° + 360° = 250°$

$-110° - 360° = -470°$

10. $\theta = -405°$

(a)



(b) Quadrant IV

(c) $-405° + 720° = 315°$

$-405° + 360° = -45°$

11. $480° = 480° \cdot \dfrac{\pi \text{ rad}}{180°}$

$= \dfrac{8\pi}{3}$ radians

≈ 8.378 radians

12. $-127.5° \cdot \dfrac{\pi}{180°} \approx -2.225$

13. $-33°\,45' = -33.75° = -33.75° \cdot \dfrac{\pi \text{ rad}}{180°}$

$= -\dfrac{3\pi}{16}$ radian ≈ -0.589 radian

14. $196°\,77' = \left(196 + \dfrac{77}{60}\right)° \cdot \dfrac{\pi}{180°} \approx 3.443$

15. $\dfrac{5\pi \text{ rad}}{7} = \dfrac{5\pi \text{ rad}}{7} \cdot \dfrac{180°}{\pi \text{ rad}} \approx 128.571°$

16. $-\dfrac{11\pi}{6} \cdot \dfrac{180°}{\pi} = -330.000°$

17. $-3.5 \text{ rad} = -3.5 \text{ rad} \cdot \dfrac{180°}{\pi \text{ rad}} \approx -200.535°$

18. $5.7 \cdot \dfrac{180°}{\pi} \approx 326.586°$

19. $138° = \dfrac{138\pi}{180} = \dfrac{23\pi}{30}$ radians

$s = r\theta = 20\left(\dfrac{23\pi}{30}\right) \approx 48.17$ inches

20. $60° = \dfrac{60\pi}{180}$ radians

$s = r\theta = 11 \cdot \left(\dfrac{60}{180}\right)\pi$

$= \dfrac{11}{3}\pi$ meters

$s \approx 11.52$ meters

21. (a) Angular speed $= \dfrac{\left(33\frac{1}{3}\right)(2\pi) \text{ radians}}{1 \text{ minute}}$

$= 66\frac{2}{3}\pi$ radians per minute

(b) Linear speed $= \dfrac{6\left(66\frac{2}{3}\pi\right) \text{ inches}}{1 \text{ minute}}$

$= 400\pi$ inches per minute

22. (linear speed) = (angular speed) · (radius)

$= (5\pi \text{ rad}/s) \cdot (13.5 \text{ inches})$

$= 67.5\pi$ inches per second

≈ 212.1 inches per second

≈ 12.05 miles per hour

23. $120° = \dfrac{120\pi}{180} = \dfrac{2\pi}{3}$ radians

$A = \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(18)^2\left(\dfrac{2\pi}{3}\right) \approx 339.29$ square inches

24. $A = \dfrac{1}{2}\theta r^2 = \dfrac{1}{2}\left(\dfrac{5\pi}{6}\right)6.5^2$

$A \approx 55.31$ square millimeters

25. $t = \dfrac{2\pi}{3}$ corresponds to the point $\left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$.

26. $t = \dfrac{3\pi}{4}$, $(x, y) = \left(-\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$

27. $t = \frac{5\pi}{6}$ corresponds to the point $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

28. $t = -\frac{4\pi}{3}, (x, y) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

29. $t = \frac{7\pi}{6}$ corresponds to the point $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

$\sin\frac{7\pi}{6} = y = -\frac{1}{2}$ $\quad \csc\frac{7\pi}{6} = \frac{1}{y} = -2$

$\cos\frac{7\pi}{6} = x = -\frac{\sqrt{3}}{2}$ $\quad \sec\frac{7\pi}{6} = \frac{1}{x} = -\frac{2\sqrt{3}}{3}$

$\tan\frac{7\pi}{6} = \frac{y}{x} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\quad \cot\frac{7\pi}{6} = \frac{x}{y} = \sqrt{3}$

30. $t = \frac{\pi}{4}$ corresponds to the point $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$\sin\frac{\pi}{4} = y = \frac{\sqrt{2}}{2}$ $\quad \csc\frac{\pi}{4} = \frac{1}{y} = \sqrt{2}$

$\cos\frac{\pi}{4} = x = \frac{\sqrt{2}}{2}$ $\quad \sec\frac{\pi}{4} = \frac{1}{x} = \sqrt{2}$

$\tan\frac{\pi}{4} = \frac{y}{x} = 1$ $\quad \cot\frac{\pi}{4} = \frac{x}{y} = 1$

31. $t = -\frac{2\pi}{3}$ corresponds to the point $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$\sin\left(-\frac{2\pi}{3}\right) = y = -\frac{\sqrt{3}}{2}$ $\quad \csc\left(-\frac{2\pi}{3}\right) = \frac{1}{y} = -\frac{2\sqrt{3}}{3}$

$\cos\left(-\frac{2\pi}{3}\right) = x = -\frac{1}{2}$ $\quad \sec\left(-\frac{2\pi}{3}\right) = \frac{1}{x} = -2$

$\tan\left(-\frac{2\pi}{3}\right) = \frac{y}{x} = \sqrt{3}$ $\quad \cot\left(-\frac{2\pi}{3}\right) = \frac{x}{y} = \frac{\sqrt{3}}{3}$

32. $t = 2\pi$ corresponds to the point $(1, 0)$.

$\sin 2\pi = y = 0$ $\quad \csc 2\pi = \frac{1}{y}$ is undefined.

$\cos 2\pi = x = 1$ $\quad \sec 2\pi = \frac{1}{x} = 1$

$\tan 2\pi = \frac{y}{x} = 0$ $\quad \cot 2\pi = \frac{x}{y}$ is undefined.

33. $\sin\frac{11\pi}{4} = \sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

34. $\cos 4\pi = \cos 0 = 1$

35. $\sin\left(-\frac{17\pi}{6}\right) = \sin\left(-\frac{5\pi}{6}\right) = -\frac{1}{2}$

36. $\cos\left(-\frac{13\pi}{3}\right) = \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$

37. $\tan 33 \approx -75.3130$

38. $\csc 10.5 = \frac{1}{\sin 10.5} \approx -1.1368$

39. $\sec\left(\frac{12\pi}{5}\right) = \frac{1}{\cos\left(\frac{12\pi}{5}\right)} \approx 3.2361$

40. $\sin\left(-\frac{\pi}{9}\right) \approx -0.3420$

41. opp $= 4$, adj $= 5$, hyp $= \sqrt{4^2 + 5^2} = \sqrt{41}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{\sqrt{41}}{4}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$ $\quad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{41}}{5}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{5}$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{5}{4}$

42. adj $= 6$, opp $= 6$

hyp $= \sqrt{6^2 + 6^2} = 6\sqrt{2}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{6}{6\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{6\sqrt{2}}{6} = \sqrt{2}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{6}{6\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\quad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{6\sqrt{2}}{6} = \sqrt{2}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{6}{6} = 1$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{6}{6} = 1$

43. adj $= 4$, hyp $= 8$, opp $= \sqrt{8^2 - 4^2} = \sqrt{48} = 4\sqrt{3}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2}$ $\quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{8}{4\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{4}{8} = \frac{1}{2}$ $\quad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{8}{4} = 2$

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4\sqrt{3}}{4} = \sqrt{3}$ $\quad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{4}{4\sqrt{3}} = \frac{\sqrt{3}}{3}$

44. opp $= 5$, hyp $= 9$

$$\text{adj} = \sqrt{9^2 - 5^2} = 2\sqrt{14}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{5}{9}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{2\sqrt{14}}{9}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{5}{2\sqrt{14}} = \frac{5\sqrt{14}}{28}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{9}{5}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{9}{2\sqrt{14}} = \frac{9\sqrt{14}}{28}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{2\sqrt{14}}{5}$$

45. $\sin\theta = \dfrac{1}{3}$

(a) $\csc\theta = \dfrac{1}{\sin\theta} = 3$

(b) $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{1}{9}$$

$$\cos^2\theta = \frac{8}{9}$$

$$\cos\theta = \sqrt{\frac{8}{9}}$$

$$\cos\theta = \frac{2\sqrt{2}}{3}$$

(c) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{3}{2\sqrt{2}} = \dfrac{3\sqrt{2}}{4}$

(d) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{1/3}{(2\sqrt{2})/3} = \dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$

46. $\tan\theta = 4$

(a) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{4}$

(b) $\sec\theta = \sqrt{1 + \tan^2\theta} = \sqrt{1 + 16} = \sqrt{17}$

(c) $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{\sqrt{17}} = \dfrac{\sqrt{17}}{17}$

(d) $\csc\theta = \sqrt{1 + \cot^2\theta} = \sqrt{1 + \dfrac{1}{16}} = \dfrac{\sqrt{17}}{4}$

47. $\csc\theta = 4$

(a) $\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{4}$

(b) $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{1}{4}\right)^2 + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{1}{16}$$

$$\cos^2\theta = \frac{15}{16}$$

$$\cos\theta = \sqrt{\frac{15}{16}}$$

$$\cos\theta = \frac{\sqrt{15}}{4}$$

(c) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{4}{\sqrt{15}} = \dfrac{4\sqrt{15}}{15}$

(d) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{1/4}{\sqrt{15}/4} = \dfrac{1}{\sqrt{15}} = \dfrac{\sqrt{15}}{15}$

48. $\csc\theta = 5$

(a) $\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{5}$

(b) $\cot\theta = \sqrt{\csc^2\theta - 1} = \sqrt{25 - 1} = 2\sqrt{6}$

(c) $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{1}{2\sqrt{6}} = \dfrac{\sqrt{6}}{12}$

(d) $\sec(90° - \theta) = \csc\theta = 5$

49. $\tan 33° \approx 0.6494$

50. $\csc 11° = \dfrac{1}{\sin 11°} \approx 5.2408$

51. $\sin 34.2° \approx 0.5621$

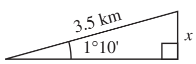
52. $\sec 79.3° = \dfrac{1}{\cos 79.3°} \approx 5.3860$

53. $\cot 15° 14' = \dfrac{1}{\tan\left(15 + \frac{14}{60}\right)} \approx 3.6722$

54. $\cos 78° 11' 58'' = \cos\left(78 + \dfrac{11}{60} + \dfrac{58}{3600}\right)° \approx 0.2045$

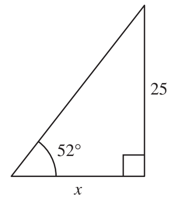
55. $\sin 1° 10' = \dfrac{x}{3.5}$

$x = 3.5 \sin 1° 10' \approx 0.07$ kilometer or 71.3 meters



Not drawn to scale

56.



$\tan 52° = \dfrac{25}{x}$

$x = \dfrac{25}{\tan 52°} \approx 19.5$ feet

57. $x = 12, y = 16, r = \sqrt{144 + 256} = \sqrt{400} = 20$

$\sin\theta = \dfrac{y}{r} = \dfrac{4}{5}$ $\quad\csc\theta = \dfrac{r}{y} = \dfrac{5}{4}$

$\cos\theta = \dfrac{x}{r} = \dfrac{3}{5}$ $\quad\sec\theta = \dfrac{r}{x} = \dfrac{5}{3}$

$\tan\theta = \dfrac{y}{x} = \dfrac{4}{3}$ $\quad\cot\theta = \dfrac{x}{y} = \dfrac{3}{4}$

58. $(x, y) = (3, -4)$

$r = \sqrt{3^2 + (-4)^2} = 5$

$\sin\theta = \dfrac{y}{r} = -\dfrac{4}{5}$ $\quad\csc\theta = \dfrac{r}{y} = -\dfrac{5}{4}$

$\cos\theta = \dfrac{x}{r} = \dfrac{3}{5}$ $\quad\sec\theta = \dfrac{r}{x} = \dfrac{5}{3}$

$\tan\theta = \dfrac{y}{x} = -\dfrac{4}{3}$ $\quad\cot\theta = \dfrac{x}{y} = -\dfrac{3}{4}$

59. $x = \dfrac{2}{3}, y = \dfrac{5}{2}$

$r = \sqrt{\left(\dfrac{2}{3}\right)^2 + \left(\dfrac{5}{2}\right)^2} = \dfrac{\sqrt{241}}{6}$

$\sin\theta = \dfrac{y}{r} = \dfrac{5/2}{\sqrt{241}/6} = \dfrac{15}{\sqrt{241}} = \dfrac{15\sqrt{241}}{241}$ $\quad\csc\theta = \dfrac{r}{y} = \dfrac{\sqrt{241}/6}{5/2} = \dfrac{2\sqrt{241}}{30} = \dfrac{\sqrt{241}}{15}$

$\cos\theta = \dfrac{x}{r} = \dfrac{2/3}{\sqrt{241}/6} = \dfrac{4}{\sqrt{241}} = \dfrac{4\sqrt{241}}{241}$ $\quad\sec\theta = \dfrac{r}{x} = \dfrac{\sqrt{241}/6}{2/3} = \dfrac{\sqrt{241}}{4}$

$\tan\theta = \dfrac{y}{x} = \dfrac{5/2}{2/3} = \dfrac{15}{4}$ $\quad\cot\theta = \dfrac{x}{y} = \dfrac{2/3}{5/2} = \dfrac{4}{15}$

60. $(x, y) = \left(-\dfrac{10}{3}, -\dfrac{2}{3}\right)$

$r = \sqrt{\left(-\dfrac{10}{3}\right)^2 + \left(-\dfrac{2}{3}\right)^2} = \dfrac{2\sqrt{26}}{3}$

$\sin\theta = \dfrac{y}{r} = \dfrac{-2/3}{\left(2\sqrt{26}\right)/3} = -\dfrac{\sqrt{26}}{26}$ $\quad\csc\theta = \dfrac{r}{y} = \dfrac{\left(2\sqrt{26}\right)/3}{-2/3} = -\sqrt{26}$

$\cos\theta = \dfrac{x}{r} = \dfrac{-10/3}{\left(2\sqrt{26}\right)/3} = -\dfrac{5\sqrt{26}}{26}$ $\quad\sec\theta = \dfrac{r}{x} = \dfrac{\left(2\sqrt{26}\right)/3}{-10/3} = -\dfrac{\sqrt{26}}{5}$

$\tan\theta = \dfrac{y}{x} = \dfrac{-2/3}{-10/3} = \dfrac{1}{5}$ $\quad\cot\theta = \dfrac{x}{y} = \dfrac{-10/3}{-2/3} = 5$

61. $x = -0.5, y = 4.5$

$$r = \sqrt{(-0.5)^2 + (4.5)^2} = \sqrt{20.5} = \frac{\sqrt{82}}{2}$$

$$\sin\theta = \frac{y}{r} = \frac{4.5}{\sqrt{82}/2} = \frac{9\sqrt{82}}{82} \qquad \csc\theta = \frac{r}{y} = \frac{\sqrt{82}/2}{4.5} = \frac{\sqrt{82}}{9}$$

$$\cos\theta = \frac{x}{r} = \frac{-0.5}{\sqrt{82}/2} = \frac{-\sqrt{82}}{82} \qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{82}/2}{-0.5} = -\sqrt{82}$$

$$\tan\theta = \frac{y}{x} = \frac{4.5}{-0.5} = -9 \qquad \cot\theta = \frac{x}{y} = \frac{-0.5}{4.5} = -\frac{1}{9}$$

62. $(x, y) = (0.3, 0.4)$

$$r = \sqrt{(0.3)^2 + (0.4)^2} = 0.5$$

$$\sin\theta = \frac{y}{r} = \frac{0.4}{0.5} = \frac{4}{5} = 0.8 \qquad \csc\theta = \frac{r}{y} = \frac{0.5}{0.4} = \frac{5}{4} = 1.25$$

$$\cos\theta = \frac{x}{r} = \frac{0.3}{0.5} = \frac{3}{5} = 0.6 \qquad \sec\theta = \frac{r}{x} = \frac{0.5}{0.3} = \frac{5}{3} \approx 1.67$$

$$\tan\theta = \frac{y}{x} = \frac{0.4}{0.3} = \frac{4}{3} \approx 1.33 \qquad \cot\theta = \frac{x}{y} = \frac{0.3}{0.4} = \frac{3}{4} = 0.75$$

63. $(x, 4x),\ x > 0$

$x' = x, y' = 4x$

$$r = \sqrt{x^2 + (4x)^2} = \sqrt{17}x$$

$$\sin\theta = \frac{y'}{r} = \frac{4x}{\sqrt{17}x} = \frac{4\sqrt{17}}{17} \qquad \csc\theta = \frac{r}{y'} = \frac{\sqrt{17}x}{4x} = \frac{\sqrt{17}}{4}$$

$$\cos\theta = \frac{x'}{r} = \frac{x}{\sqrt{17}x} = \frac{\sqrt{17}}{17} \qquad \sec\theta = \frac{r}{x'} = \frac{\sqrt{17}x}{x} = \sqrt{17}$$

$$\tan\theta = \frac{y'}{x'} = \frac{4x}{x} = 4 \qquad \cot\theta = \frac{x'}{y'} = \frac{x}{4x} = \frac{1}{4}$$

64. $(x', y') = (-2x, -3x), x > 0$

$$r = \sqrt{(-2x)^2 + (-3x)^2} = \sqrt{13}x$$

$$\sin\theta = \frac{y'}{r} = \frac{-3x}{\sqrt{13}x} = -\frac{3\sqrt{13}}{13}$$

$$\cos\theta = \frac{x'}{r} = \frac{-2x}{\sqrt{13}x} = -\frac{2\sqrt{13}}{13}$$

$$\tan\theta = \frac{y'}{x'} = \frac{-3x}{-2x} = \frac{3}{2}$$

$$\csc\theta = \frac{r}{y'} = \frac{\sqrt{13}x}{-3x} = -\frac{\sqrt{13}}{3}$$

$$\sec\theta = \frac{r}{x'} = \frac{\sqrt{13}x}{-2x} = -\frac{\sqrt{13}}{2}$$

$$\cot\theta = \frac{x'}{y'} = \frac{-2x}{-3x} = \frac{2}{3}$$

65. $\sec\theta = \frac{6}{5},\ \tan\theta < 0 \implies \theta$ is in Quadrant IV.

$$r = 6, x = 5, y = -\sqrt{36 - 25} = -\sqrt{11}$$

$$\sin\theta = \frac{y}{r} = -\frac{\sqrt{11}}{6}$$

$$\cos\theta = \frac{x}{r} = \frac{5}{6}$$

$$\tan\theta = \frac{y}{x} = -\frac{\sqrt{11}}{5}$$

$$\csc\theta = \frac{r}{y} = -\frac{6\sqrt{11}}{11}$$

$$\sec\theta = \frac{6}{5}$$

$$\cot\theta = -\frac{5\sqrt{11}}{11}$$

66. $\csc\theta = \frac{3}{2}, \cos\theta < 0$

θ is in Quadrant II.

$\sin\theta = \frac{1}{\csc\theta} = \frac{2}{3}$

$\cos\theta = -\sqrt{1-\sin^2\theta} = -\frac{\sqrt{5}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{2\sqrt{5}}{5}$

$\sec\theta = \frac{1}{\cos\theta} = -\frac{3\sqrt{5}}{5}$

$\cot\theta = \frac{1}{\tan\theta} = -\frac{\sqrt{5}}{2}$

67. $\sin\theta = \frac{3}{8}, \cos\theta < 0 \implies \theta$ is in Quadrant II.

$y = 3, r = 8, x = -\sqrt{55}$

$\sin\theta = \frac{y}{r} = \frac{3}{8}$

$\cos\theta = \frac{x}{r} = -\frac{\sqrt{55}}{8}$

$\tan\theta = \frac{y}{x} = -\frac{3}{\sqrt{55}} = -\frac{3\sqrt{55}}{55}$

$\csc\theta = \frac{8}{3}$

$\sec\theta = -\frac{8}{\sqrt{55}} = -\frac{8\sqrt{55}}{55}$

$\cot\theta = -\frac{\sqrt{55}}{3}$

68. $\tan\theta = \frac{5}{4}, \cos\theta < 0$

θ is in Quadrant III.

$\sec\theta = -\sqrt{1+\tan^2\theta} = -\sqrt{1+\left(\frac{25}{16}\right)} = -\frac{\sqrt{41}}{4}$

$\cos\theta = \frac{1}{\sec\theta} = -\frac{4\sqrt{41}}{41}$

$\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{16}{41}} = -\frac{5\sqrt{41}}{41}$

$\csc\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{41}}{5}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{4}{5}$

69. $\cos\theta = \frac{x}{r} = \frac{-2}{5} \implies y^2 = 21$

$\sin\theta > 0 \implies \theta$ is in Quadrant II $\implies y = \sqrt{21}$

$\sin\theta = \frac{y}{r} = \frac{\sqrt{21}}{5}$

$\tan\theta = \frac{y}{x} = -\frac{\sqrt{21}}{2}$

$\csc\theta = \frac{r}{y} = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$

$\sec\theta = \frac{r}{x} = \frac{5}{-2} = -\frac{5}{2}$

$\cot\theta = \frac{x}{y} = \frac{-2}{\sqrt{21}} = -\frac{2\sqrt{21}}{21}$

70. $\sin\theta = -\frac{2}{4} = -\frac{1}{2}, \cos\theta > 0$

θ is in Quadrant IV.

$\csc\theta = \frac{1}{\sin\theta} = -2$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{1}{4}\right)} = \frac{\sqrt{3}}{2}$

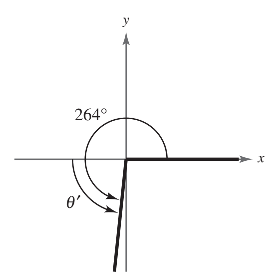
$\sec\theta = \frac{1}{\cos\theta} = \frac{2\sqrt{3}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{\sqrt{3}}{3}$

$\cot\theta = \frac{1}{\tan\theta} = -\sqrt{3}$

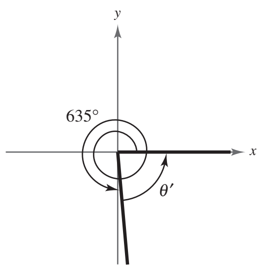
71. $\theta = 264°$

$\theta' = 264° - 180° = 84°$



72. $\theta = 635° = 720° - 85°$

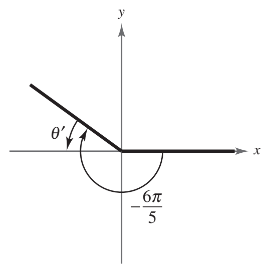
$\theta' = 85°$



73. $\theta = -\dfrac{6\pi}{5}$

$-\dfrac{6\pi}{5} + 2\pi = \dfrac{4\pi}{5}$

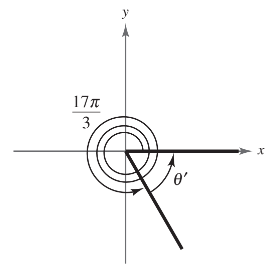
$\theta' = \pi - \dfrac{4\pi}{5} = \dfrac{\pi}{5}$



74. $\theta = \dfrac{17\pi}{3} = \dfrac{18\pi}{3} - \dfrac{\pi}{3}$

$= 6\pi - \dfrac{\pi}{3}$

$\theta' = \dfrac{\pi}{3}$



75. $\sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

$\cos\dfrac{\pi}{3} = \dfrac{1}{2}$

$\tan\dfrac{\pi}{3} = \sqrt{3}$

76. $\sin\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

$\cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

$\tan\dfrac{\pi}{4} = \dfrac{2}{\sqrt{2}/2} = 1$

77. $\sin\left(-\dfrac{7\pi}{3}\right) = -\sin\dfrac{\pi}{3} = -\dfrac{\sqrt{3}}{2}$

$\cos\left(-\dfrac{7\pi}{3}\right) = \cos\dfrac{\pi}{3} = \dfrac{1}{2}$

$\tan\left(-\dfrac{7\pi}{3}\right) = -\tan\dfrac{\pi}{3} = -\sqrt{3}$

78. $\sin\left(-\dfrac{5\pi}{4}\right) = \sin\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

$\cos\left(-\dfrac{5\pi}{4}\right) = -\cos\dfrac{\pi}{4} = -\dfrac{\sqrt{2}}{2}$

$\tan\left(-\dfrac{5\pi}{4}\right) = -\tan\dfrac{\pi}{4}$

$= \dfrac{2}{-\sqrt{2}/2} = -1$

79. $\sin 495° = \sin 45° = \dfrac{\sqrt{2}}{2}$

$\cos 495° = -\cos 45° = -\dfrac{\sqrt{2}}{2}$

$\tan 495° = -\tan 45° = -1$

80. $\sin(-150°) = -\dfrac{1}{2}$

$\cos(-150°) = -\dfrac{\sqrt{3}}{2}$

$\tan(-150°) = \dfrac{-1/2}{-\sqrt{3}/2} = \dfrac{\sqrt{3}}{3}$

81. $\sin(-240°) = \sin 60° = \dfrac{\sqrt{3}}{2}$

$\cos(-240°) = -\cos 60° = -\dfrac{1}{2}$

$\tan(-240°) = -\tan 60° = -\sqrt{3}$

82. $\sin(315°) = -\dfrac{\sqrt{2}}{2}$

$\cos(315°) = \dfrac{\sqrt{2}}{2}$

$\tan(315°) = \dfrac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$

83. $\sin 4 \approx -0.7568$

84. $\tan 3 \approx -0.1425$

85. $\sin(-3.2) \approx 0.0584$

86. $\cot(-4.8) = \dfrac{1}{\tan(-4.8)} \approx 0.0878$

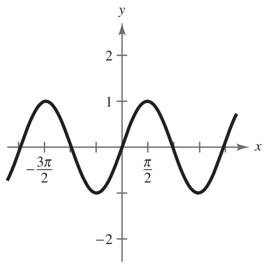
87. $\sec\left(\dfrac{12\pi}{5}\right) = \dfrac{1}{\cos\left(\dfrac{12\pi}{5}\right)} \approx 3.2361$

88. $\tan\left(\dfrac{-25\pi}{7}\right) \approx 4.3813$

89. $y = \sin x$

Amplitude: 1

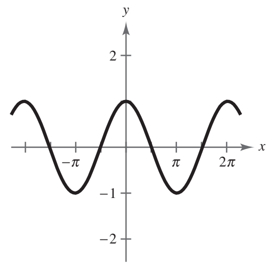
Period: 2π



90. $y = \cos x$

Amplitude: 1

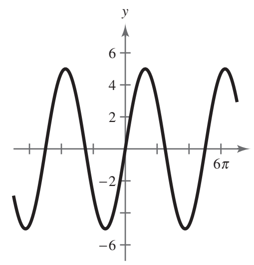
Period: 2π



91. $f(x) = 5 \sin \dfrac{2x}{5}$

Amplitude: 5

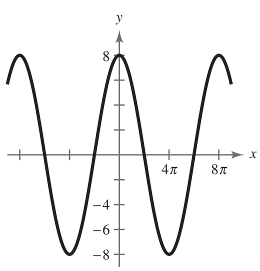
Period: $\dfrac{2\pi}{2/5} = 5\pi$



92. $f(x) = 8 \cos\left(-\dfrac{x}{4}\right)$

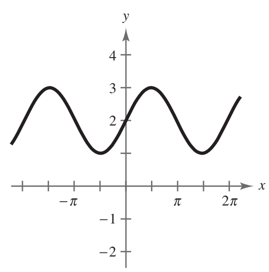
Amplitude: 8

Period: $\dfrac{2\pi}{1/4} = 8\pi$



93. $y = 2 + \sin x$

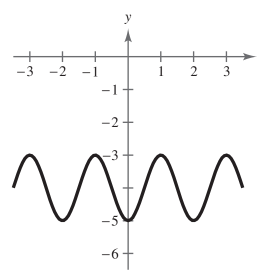
Shift the graph of $y = \sin x$ two units upward.



94. $y = -4 - \cos \pi x$

Amplitude: 1

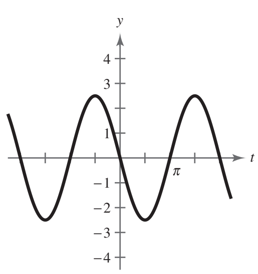
Period: $\dfrac{2\pi}{\pi} = 2$



95. $g(t) = \dfrac{5}{2} \sin(t - \pi)$

Amplitude: $\dfrac{5}{2}$

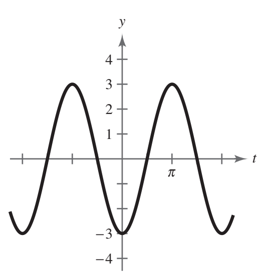
Period: 2π



96. $g(t) = 3 \cos(t + \pi)$

Amplitude: 3

Period: 2π



97. $y = a \sin bx$

(a) $a = 2$,

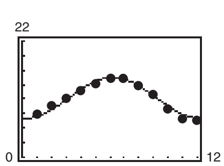
$$\frac{2\pi}{b} = \frac{1}{264} \implies b = 528\pi$$

$$y = 2 \sin(528\pi x)$$

(b) $f = \dfrac{1}{1/264}$

$= 264$ cycles per second.

98. (a) $S(t) = 18.09 + 1.41 \sin\left(\dfrac{\pi t}{6} + 4.60\right)$



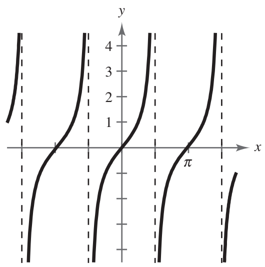
(b) Period $= \dfrac{2\pi}{\pi/6} = (2)(6) = 12$

12 months = 1 year, so this is expected.

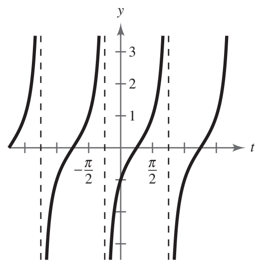
(c) Amplitude: 1.41

The amplitude represents the maximum change in the time of sunset from the average time ($d = 18.09$).

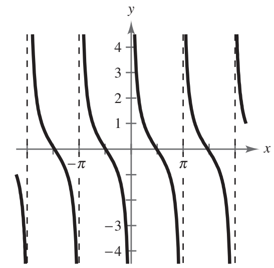
99. $f(x) = \tan x$



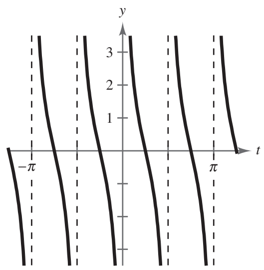
100. $f(t) = \tan\left(t - \dfrac{\pi}{4}\right)$



101. $f(x) = \cot x$

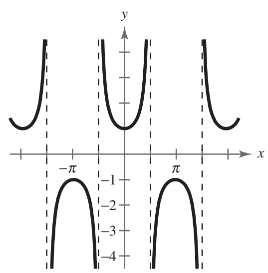


102. $g(t) = 2 \cot 2t$

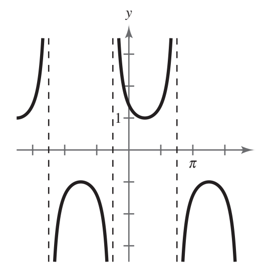


103. $f(x) = \sec x$

Graph $y = \cos x$ first.

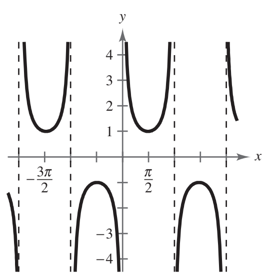


104. $h(t) = \sec\left(t - \dfrac{\pi}{4}\right)$

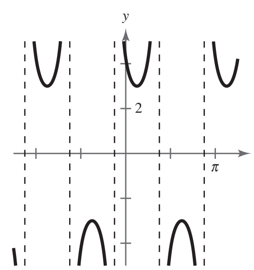


105. $f(x) = \csc x$

Graph $y = \sin x$ first.



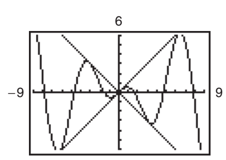
106. $f(t) = 3 \csc\left(2t + \dfrac{\pi}{4}\right)$



107. $f(x) = x \cos x$

Graph $y = x$ and $y = -x$ first.

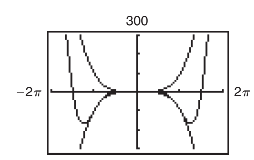
As $x \to \infty, f(x) \to \infty$.



108. $g(x) = x^4 \cos x$

Damping factor: x^4

As $x \to \infty, f(x) \to \infty$.



109. $\arcsin\left(-\dfrac{1}{2}\right) = -\arcsin \dfrac{1}{2} = -\dfrac{\pi}{6}$

110. $\arcsin(-1) = -\dfrac{\pi}{2}$

111. $\arcsin 0.4 \approx 0.41$ radian

112. $\arcsin(0.213) \approx 0.21$ radian

113. $\sin^{-1}(-0.44) \approx -0.46$ radian

114. $\sin^{-1}(0.89) \approx 1.10$ radians

115. $\arccos \dfrac{\sqrt{3}}{2} = \dfrac{\pi}{6}$

116. $\arccos\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi}{4}$

117. $\cos^{-1}(-1) = \pi$

118. $\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{6}$

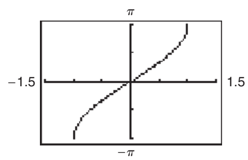
119. $\arccos 0.324 \approx 1.24$ radians

120. $\arccos(-0.888) \approx 2.66$ radians

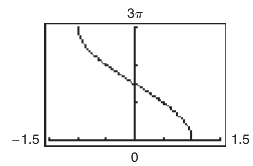
121. $\tan^{-1}(-1.5) \approx -0.98$ radian

122. $\tan^{-1}(8.2) \approx 1.45$ radians

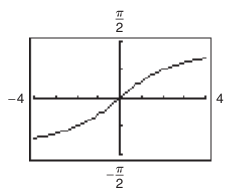
123. $f(x) = 2 \arcsin x = 2 \sin^{-1}(x)$



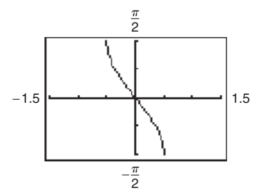
124. $y = 3 \arccos x$



125. $f(x) = \arctan\left(\dfrac{x}{2}\right) = \tan^{-1}\left(\dfrac{x}{2}\right)$

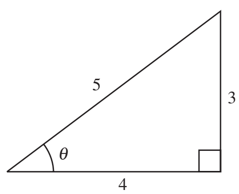


126. $f(x) = -\arcsin 2x$



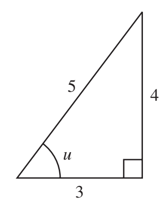
127. $\cos\left(\arctan \frac{3}{4}\right) = \frac{4}{5}$

Use a right triangle. Let $\theta = \arctan \frac{3}{4}$ then $\tan \theta = \frac{3}{4}$ and $\cos \theta = \frac{4}{5}$.



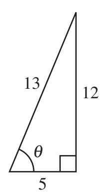
128. Let $u = \arccos \frac{3}{5}$.

$\tan\left(\arccos \frac{3}{5}\right) = \tan u = \frac{4}{3}$



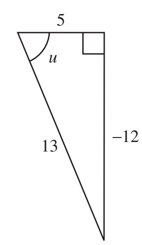
129. $\sec\left(\arctan \frac{12}{5}\right) = \frac{13}{5}$

Use a right triangle. Let $\theta = \arctan \frac{12}{5}$ then $\tan \theta = \frac{12}{5}$ and $\sec \theta = \frac{13}{5}$.



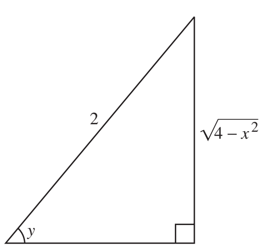
130. Let $u = \arcsin\left(-\frac{12}{13}\right)$.

$\cot\left[\arcsin\left(-\frac{12}{13}\right)\right] = \cot u = -\frac{5}{12}$



131. Let $y = \arccos\left(\dfrac{x}{2}\right)$. Then

$\cos y = \dfrac{x}{2}$ and $\tan y = \tan\left(\arccos\left(\dfrac{x}{2}\right)\right) = \dfrac{\sqrt{4 - x^2}}{x}$.



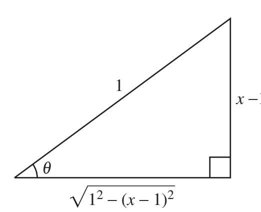
132. $\sec(\arcsin(x - 1))$

$\theta = \arcsin(x - 1) \implies -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$

$\sin \theta = x - 1$

$\cos \theta = \sqrt{1^2 - (x - 1)^2} = \sqrt{x(2 - x)}$

$\sec \theta = \dfrac{1}{\sqrt{x(2 - x)}}$

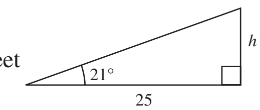


133. $\tan \theta = \dfrac{70}{30}$

$\theta = \arctan\left(\dfrac{70}{30}\right) \approx 66.8°$

134. $\tan 21° = \dfrac{h}{25}$

$h = 25 \tan 21° \approx 9.6$ feet



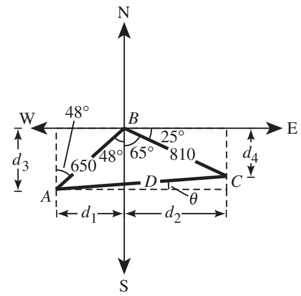
135.
$$\left.\begin{aligned} \sin 48° &= \frac{d_1}{650} \implies d_1 \approx 483 \\ \cos 25° &= \frac{d_2}{810} \implies d_2 \approx 734 \end{aligned}\right\} \quad d_1 + d_2 \approx 1217$$

$$\left.\begin{aligned} \cos 48° &= \frac{d_3}{650} \implies d_3 \approx 435 \\ \sin 25° &= \frac{d_4}{810} \implies d_4 \approx 342 \end{aligned}\right\} \quad d_3 - d_4 \approx 93$$

$\tan \theta \approx \dfrac{93}{1217} \implies \theta \approx 4.4°$

$\sec 4.4° \approx \dfrac{D}{1217} \implies D \approx 1217 \sec 4.4° \approx 1221$

The distance is 1221 miles and the bearing is 85.6°.



136. Amplitude: $\dfrac{1.5}{2} = 0.75$ inches

Period: 3 seconds

$d = a \cos bt$

$a = 0.75$

$b = \dfrac{2\pi}{3}$

$d = 0.75 \cos\left(\dfrac{2\pi t}{3}\right)$

137. False. The sine or cosine functions are often useful for modeling simple harmonic motion.

138. True. The inverse sine, $y = \arcsin x$, is defined where $-1 \le x \le 1$ and $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2}$.

139. False. For each θ there corresponds exactly one value of y.

140. False. The range of arctan is $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$, so $\arctan(-1) = -\dfrac{\pi}{4}$.

141. $y = 3 \sin x$

Amplitude: 3

Period: 2π

Matches graph (d)

142. $y = -3 \sin x$ matches graph (a).

Period: 2π

Amplitude: 3

143. $y = 2 \sin \pi x$

Amplitude: 2

Period: 2

Matches graph (b)

144. $y = 2 \sin \dfrac{x}{2}$ matches graph (c).

Period: 4π

Amplitude: 2

145. $f(\theta) = \sec \theta$ is undefined at the zeros of $g(\theta) = \cos \theta$ since $\sec \theta = \dfrac{1}{\cos \theta}$.

146. (a)

θ	0.1	0.4	0.7	1.0	1.3
$\tan\left(\theta - \frac{\pi}{2}\right)$	-9.9666	-2.3652	-1.1872	-0.6421	-0.2776
$-\cot\theta$	-9.9666	-2.3652	-1.1872	-0.6421	-0.2776

(b) $\tan\left(\theta - \frac{\pi}{2}\right) = -\cot\theta$

147. The ranges for the other four trigonometric functions are not bounded. For $y = \tan x$ and $y = \cot x$, the range is $(-\infty, \infty)$. For $y = \sec x$ and $y = \csc x$, the range is $(-\infty, -1] \cup [1, \infty)$.

148. $y = Ae^{-kt}\cos bt = \frac{1}{5}e^{-t/10}\cos 6t$

(a) A is changed from $\frac{1}{5}$ to $\frac{1}{3}$: The displacement is increased.

(b) k is changed from $\frac{1}{10}$ to $\frac{1}{3}$: The friction damps the oscillations more rapidly.

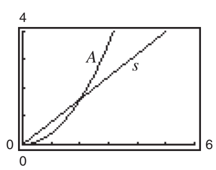
(c) b is changed from 6 to 9: The frequency of oscillation is increased.

149. $A = \frac{1}{2}r^2\theta$, $s = r\theta$

(a) $A = \frac{1}{2}r^2(0.8) = 0.4r^2, r > 0$

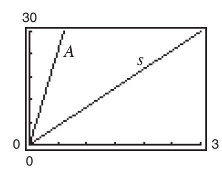
$s = r(0.8) = 0.8r, r > 0$

As r increases, the area function increases more rapidly.



(b) $A = \frac{1}{2}(10)^2\theta = 50\theta, \theta > 0$

$s = 10\theta, \theta > 0$



150. Answers will vary.

Problem Solving for Chapter 4

1. (a) $8{:}57 - 6{:}45 = 2$ hours 12 minutes $= 132$ minutes

$\frac{132}{48} = \frac{11}{4}$ revolutions

$\theta = \left(\frac{11}{4}\right)(2\pi) = \frac{11\pi}{2}$ radians or $990°$

(b) $s = r\theta = 47.25(5.5\pi) \approx 816.42$ feet

2. Gear 1: $\frac{24}{32}(360°) = 270° = \frac{3\pi}{2}$ radians

Gear 2: $\frac{24}{26}(360°) \approx 332.308° \approx 5.80$ radians

Gear 3: $\frac{24}{22}(360°) \approx 392.727° \approx 6.85$ radians

Gear 4: $\frac{40}{32}(360°) = 450° = \frac{5\pi}{2}$ radians

Gear 5: $\frac{24}{19}(360°) \approx 454.737° \approx 7.94$ radians

3. (a) $\sin 39° = \frac{3000}{d}$

$$d = \frac{3000}{\sin 39°} \approx 4767 \text{ feet}$$

(b) $\tan 39° = \frac{3000}{x}$

$$x = \frac{3000}{\tan 39°} \approx 3705 \text{ feet}$$

(c)
$$\tan 63° = \frac{w + 3705}{3000}$$
$$3000\tan 63° = w + 3705$$
$$w = 3000\tan 63° - 3705 \approx 2183 \text{ feet}$$

4. (a) $\triangle ABC$, $\triangle ADE$, and $\triangle AFG$ are all similar triangles since they all have the same angles. $\angle A$ is part of all three triangles and $\angle C = \angle E = \angle G = 90°$. Thus, $\angle B = \angle D = \angle F$.

(b) Since the triangles are similar, the ratios of corresponding sides are equal.

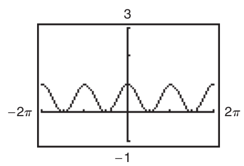
$$\frac{BC}{AB} = \frac{DE}{AD} = \frac{FG}{AF}$$

(c) Since the ratios: $\dfrac{\text{opp}}{\text{hyp}} = \dfrac{BC}{AB} = \dfrac{DE}{AD} = \dfrac{FG}{AF} = \sin A$ it does not matter which triangle is used to calculate $\sin A$.

Any triangle similar to these three triangles could be used to find $\sin A$. The value of $\sin A$ would not change.

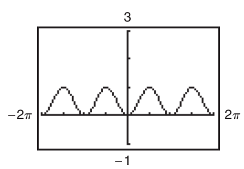
(d) Since the values of all six trigonometric functions can be found by taking the ratios of the sides of a right triangle, similar triangles would yield the same values.

5. (a) $h(x) = \cos^2 x$



h is even.

(b) $h(x) = \sin^2 x$



h is even.

6. Given: f is an even function and g is an odd function.

(a)
$$\begin{aligned} h(x) &= [f(x)]^2 \\ h(-x) &= [f(-x)]^2 \\ &= [f(x)]^2 \text{ since } f \text{ is even} \\ &= h(x) \end{aligned}$$

Thus, h is an even function.

(b)
$$\begin{aligned} h(x) &= [g(x)]^2 \\ h(-x) &= [g(-x)]^2 \\ &= [-g(x)]^2 \text{ since } g \text{ is odd} \\ &= [g(x)]^2 \\ &= h(x) \end{aligned}$$

Thus, h is an even function.

Conjecture: The square of either an even function or an odd function is an even function.

7. If we alter the model so that $h = 1$ when $t = 0$, we can use either a sine or a cosine model.

$$a = \frac{1}{2}[\max - \min] = \frac{1}{2}[101 - 1] = 50$$

$$d = \frac{1}{2}[\max + \min] = \frac{1}{2}[101 + 1] = 51$$

$$b = 8\pi$$

For the cosine model we have: $h = 51 - 50\cos(8\pi t)$

For the sine model we have: $h = 51 - 50\sin\left(8\pi t + \dfrac{\pi}{2}\right)$

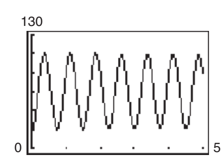
Notice that we needed the horizontal shift so that the sine value was one when $t = 0$.

Another model would be: $h = 51 + 50\sin\left(8\pi t + \dfrac{3\pi}{2}\right)$

Here we wanted the sine value to be 1 when $t = 0$.

8. $P = 100 - 20\cos\left(\dfrac{8\pi}{3}t\right)$

(a)



(b) Period $= \dfrac{2\pi}{8\pi/3} = \dfrac{6}{8} = \dfrac{3}{4}$ sec

This is the time between heartbeats.

(c) Amplitude: 20

The blood pressure ranges between $100 - 20 = 80$ and $100 + 20 = 120$.

(d) Pulse rate $= \dfrac{60 \text{ sec/min}}{\frac{3}{4}\text{ sec/beat}} = 80$ beats/min

(e) Period $= \dfrac{60}{64} = \dfrac{15}{16}$ sec

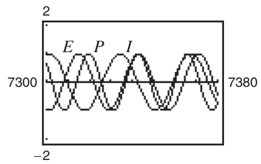
$$64 = \frac{60}{2\pi/b} \implies b = \frac{64}{60} \cdot 2\pi = \frac{32}{15}\pi$$

9. Physical (23 days): $P = \sin \frac{2\pi t}{23}, t \geq 0$

Emotional (28 days): $E = \sin \frac{2\pi t}{28}, t \geq 0$

Intellectual (33 days): $I = \sin \frac{2\pi t}{33}, t \geq 0$

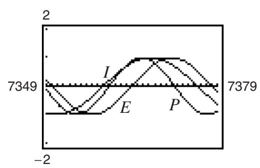
(a)



(b) Number of days since birth until September 1, 2006:

$$t = \underbrace{365 \times 20}_{\text{20 years}} + \underbrace{5}_{\text{leap years}} + \underbrace{11}_{\text{remaining July days}} + \underbrace{31}_{\text{August days}} + \underbrace{1}_{\text{day in September}}$$

$t = 7348$



All three drop early in the month, then peak toward the middle of the month, and drop again toward the latter part of the month.

(c) For September 22, 2006, use $t = 7369$.

$P \approx 0.631$

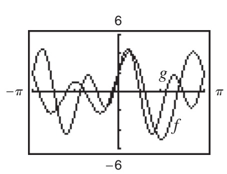
$E \approx 0.901$

$I \approx 0.945$

10. $f(x) = 2 \cos 2x + 3 \sin 3x$

$g(x) = 2 \cos 2x + 3 \sin 4x$

(a)



(b) The period of $f(x)$ is 2π.

The period of $g(x)$ is π.

(c) $h(x) = A \cos \alpha x + B \sin \beta x$ is periodic since the sine and cosine functions are periodic.

11. (a) Both graphs have a period of 2 and intersect when $x = 5.35$. They should also intersect when $x = 5.35 - 2 = 3.35$ and $x = 5.35 + 2 = 7.35$.

(b) The graphs intersect when $x = 5.35 - 3(2) = -0.65$.

(c) Since $13.35 = 5.35 + 4(2)$ and $-4.65 = 5.35 - 5(2)$ the graphs will intersect again at these values. Therefore $f(13.35) = g(-4.65)$.

12. (a) $f(t - 2c) = f(t)$ is true since this is a two period horizontal shift.

(b) $f\left(t + \frac{1}{2}c\right) = f\left(\frac{1}{2}t\right)$ is not true.

$f\left(t + \frac{1}{2}c\right)$ is a horizontal translation of $f(t)$.

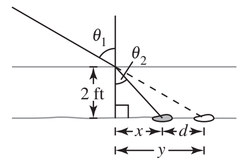
$f\left(\frac{1}{2}t\right)$ is a doubling of the period of $f(t)$.

(c) $f\left(\frac{1}{2}(t + c)\right) = f\left(\frac{1}{2}t\right)$ is not true.

$f\left(\frac{1}{2}(t + c)\right) = f\left(\frac{1}{2}t + \frac{1}{2}c\right)$ is a horizontal translation of $f\left(\frac{1}{2}t\right)$ by half a period.

For example, $\sin\left[\frac{1}{2}(\pi + 2\pi)\right] \neq \sin\left(\frac{1}{2}\pi\right)$.

13.



(a) $\dfrac{\sin\theta_1}{\sin\theta_2} = 1.333$

$\sin\theta_2 = \dfrac{\sin\theta_1}{1.333} = \dfrac{\sin 60^\circ}{1.333} \approx 0.6497$

$\theta_2 \approx 40.52^\circ$

(b) $\tan\theta_2 = \dfrac{x}{2} \implies x = 2\tan 40.52^\circ \approx 1.71$ feet

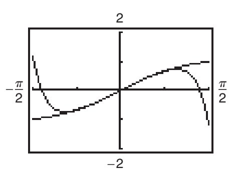
$\tan\theta_1 = \dfrac{y}{2} \implies y = 2\tan 60^\circ \approx 3.46$ feet

(c) $d = y - x = 3.46 - 1.71 = 1.75$ feet

(d) As you more closer to the rock, θ_1 decreases, which causes y to decrease, which in turn causes d to decrease.

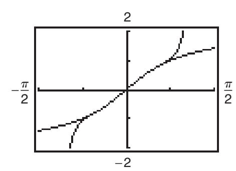
14. $\arctan x \approx x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7}$

(a)



The graphs are nearly the same for $-1 < x < 1$.

(b)



The accuracy of the approximation improved slightly by adding the next term $(x^9/9)$.

Chapter 4 Practice Test

1. Express 350° in radian measure.

2. Express $(5\pi)/9$ in degree measure.

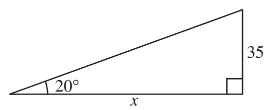
3. Convert 135° 14′ 12″ to decimal form.

4. Convert $-22.569°$ to D° M′ S″ form.

5. If $\cos\theta = \frac{2}{3}$, use the trigonometric identities to find $\tan\theta$.

6. Find θ given $\sin\theta = 0.9063$.

7. Solve for x in the figure below.



8. Find the reference angle θ' for $\theta = (6\pi)/5$.

9. Evaluate csc 3.92.

10. Find $\sec\theta$ given that θ lies in Quadrant III and $\tan\theta = 6$.

11. Graph $y = 3\sin\dfrac{x}{2}$.

12. Graph $y = -2\cos(x - \pi)$.

13. Graph $y = \tan 2x$.

14. Graph $y = -\csc\left(x + \dfrac{\pi}{4}\right)$.

15. Graph $y = 2x + \sin x$, using a graphing calculator.

16. Graph $y = 3x\cos x$, using a graphing calculator.

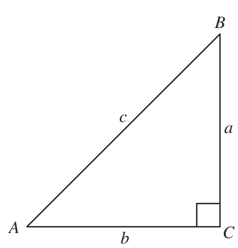
17. Evaluate arcsin 1.

18. Evaluate $\arctan(-3)$.

19. Evaluate $\sin\left(\arccos\dfrac{4}{\sqrt{35}}\right)$.

20. Write an algebraic expression for $\cos\left(\arcsin\dfrac{x}{4}\right)$.

For Exercises 21–23, solve the right triangle.



21. $A = 40°,\ c = 12$

22. $B = 6.84°,\ a = 21.3$

23. $a = 5,\ b = 9$

24. A 20-foot ladder leans against the side of a barn. Find the height of the top of the ladder if the angle of elevation of the ladder is 67°.

25. An observer in a lighthouse 250 feet above sea level spots a ship off the shore. If the angle of depression to the ship is 5°, how far out is the ship?